Trades Access Common Core

Line D: Organizational Skills

Competency D-1: Solve Trades Mathematical Problems

B.C. Open Textbook Project
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Trades Access
COMMON CORE

Line D: Organizational Skills

Competency D-1: Solve Trades Mathematical Problems

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Foreword

The BC Open Textbook Project began in 2012 with the goal of making post-secondary education in British Columbia more accessible by reducing student cost through the use of openly licensed textbooks. The BC Open Textbook Project is administered by BCcampus and is funded by the British Columbia Ministry of Advanced Education.

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Preface

The concept of identifying and creating resources for skills that are common to many trades has a long history in the Province of British Columbia. This collection of Trades Access Common Core (TACC) resources was adapted from the 15 Trades Common Core line modules co-published by the Industry Training and Apprenticeship Commission (ITAC) and the Centre for Curriculum Transfer and Technology (C2T2) in 2000-2002. Those modules were revisions of the original Common Core portion of the TRAC modules prepared by the Province of British Columbia Ministry of Post-Secondary Education in 1986. The TACC resources are still in use by a number of trades programs today and, with the permission from the Industry Training Authority (ITA), have been utilized in this project.

These open resources have been updated and realigned to match many of the line and competency titles found in the Province of BC's trades apprenticeship program outlines. A review was carried out to analyze the provincial program outlines of a number of trades, with the intent of finding common entry-level learning tasks that could be assembled into this package. This analysis provided the template for the outline used to update the existing modules. Many images found in ITA apprentice training modules were also incorporated into these resources to create books that are similar to what students will see when they continue their chosen trades training. The project team has also taken many new photographs for this project, which are available for use in other trades training resources.

The following list of lines and competencies was generated with the goal of creating an entry-level trades training resource, while still offering the flexibility for lines to be used as stand-alone books. This flexibility—in addition to the textbook content being openly licensed—allows these resources to be used within other contexts as well. For example, instructors or institutions may incorporate these resources into foundation-level trades training programming or within an online learning management system (LMS).

Line A – Safe Work Practices

- A-1 Control Workplace Hazards
- A-2 Describe WorkSafeBC Regulations
- A-3 Handle Hazardous Materials Safely
- A-4 Describe Personal Safety Practices
- A-5 Describe Fire Safety

Line B – Employability Skills

- B-1 Apply Study and Learning Skills
- B-2 Describe Expectations and Responsibilities of Employers and Employees
- B-3 Use Interpersonal Communication Skills
- B-4 Describe the Apprenticeship System

Line C – Tools and Equipment

- C-1 Describe Common Hand Tools and Their Uses
- C-2 Describe Common Power Tools and Their Uses
- C-3 Describe Rigging and Hoisting Equipment
- C-4 Describe Ladders and Platforms

Line D – Organizational Skills

- D-1 Solve Trades Mathematical Problems
- D-2 Apply Science Concepts to Trades Applications
- D-3 Read Drawings and Specifications
- D-4 Use Codes, Regulations, and Standards
- D-5 Use Manufacturer and Supplier Documentation
- D-6 Plan Projects

Line E – Electrical Fundamentals

- E-1 Describe the Basic Principles of Electricity
- E-2 Identify Common Circuit Components and Their Symbols
- E-3 Explain Wiring Connections
- E-4 Use Multimeters

All of these textbooks are available in a variety of formats in addition to print:

- PDF—printable document with TOC and hyperlinks intact
- HTML—basic export of an HTML file and its assets, suitable for use in learning management systems
- Reflowable EPUB—format that is suitable for all screen sizes including phones

All of the self-test questions are also available from BCcampus as separate data, if instructors would like to use the questions for online quizzes or competency testing.

About This Book

In an effort to make this book a flexible resource for trainers and learners, the following features are included:

- An introduction outlining the high-level goal of the Competency, and a list of objectives reflecting the skills and knowledge a person would need to achieve to fulfill this goal.
- Discrete Learning Tasks designed to help a person achieve these objectives
- Self-tests at the end of each Learning Task, designed to informally test for understanding.
- A reminder at the end of each Competency to complete a Competency test. Individual trainers are expected to determine the requirements for this test, as required.
- Throughout the textbook, there may also be links and/or references to other resources that learners will need to access, some of which are only available online.
- Notes, cautions, and warnings are identified by special symbols. A list of those symbols is provided below.

Symbols Legend

Important: This icon highlights important information.

Poisonous: This icon is a reminder for a potentially toxic/poisonous situation.

Resources: The resource icon highlights any required or optional resources.

Flammable: This icon is a reminder for a potentially flammable situation.

Self-test: This icon reminds you to complete a self-test.

Explosive: This icon is a reminder for a possibly explosive situation.

Safety gear: The safety gear icon is an important reminder to use protective equipment.

Electric shock: This icon is a reminder for potential electric shock.

Safety Advisory

Be advised that references to the Workers' Compensation Board of British Columbia safety regulations contained within these materials do not/may not reflect the most recent Occupational Health and Safety Regulation. The current Standards and Regulation in BC can be obtained at the following website: http://www.worksafebc.com.

Please note that it is always the responsibility of any person using these materials to inform him/herself about the Occupational Health and Safety Regulation pertaining to his/her area of work.

BCcampus
January 2015

Disclaimer

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Introduction

Essential skills are used in every job to varying degrees and at different levels of complexity. They provide the foundation for learning all other skills and enable people to evolve in their jobs and adapt to changes in the workplace.

Numeracy is one of the essential skills needed for the workplace. *Numeracy* refers to the ability to use numbers and the capability to think in quantitative terms. Numeracy is used when doing numerical estimating, money math, scheduling or budgeting math, and analyzing measurements or data. Trades workers also use numeracy to take a variety of measurements using basic tools, to calculate and estimate the material requirements for projects, and to compare measurements to specifications.

Although this Competency focusses on the essential skill of numeracy, other essential skills such as reading text, using documents, and thinking are practised in the process of completing this competency.

Objectives

When you have completed the Learning Tasks in this Competency, you should be able to:

- Use arithmetic and estimation
- Work with measurements
- Work with signed numbers and algebraic expressions
- Solve problems involving ratio and proportion
- Solve problems involving percent
- Solve problems involving powers, roots, and scientific notation
- Solve problems involving perimeters, areas, and volumes
- Solve problems involving geometric shapes

Resources

R You will be required to reference publications and videos available online.

LEARNING TASK 1

Use arithmetic and estimation

Being able to work with whole numbers, decimals, and fractions is important in everyday life. Estimating an answer to a problem helps put the problem in context. This process should be part of any problem-solving activity. Checking answers and solutions to problems is equally important. Work through the examples and self-test for this Learning Task without the use of a calculator.

Key terms

Common factor: A number that divides evenly into both the numerator and the denominator of a fraction.

Denominator: The number in a fraction below the line indicating how many parts the whole is divided into.

Difference: The answer to a subtraction question.

Equivalent: Quantities that are equal.

Factor: A number that divides evenly into another number; for example, 2 and 3 are factors of 6.

Improper fraction: A fraction where the numerator is the same size as or larger than the denominator; for example, 8/3, 4/4, and 9/5.

Like fractions: Fractions that have the same denominator (usually called a *common denominator*); for example, 7/8 and 3/8.

Lowest common denominator (LCD): The smallest whole number that contains the denominators of **unlike fractions**; for example, for the denominators 4 and 6, the lowest common denominator is 12.

Mixed number: A number consisting of a whole number and a fraction; for example, 2⅔, 7¾, 5⁹⁄₁₆.

Numerator: The number in a fraction above the line, indicating how many equal parts there are.

Prime number: A number that is divisible only by itself and 1; for example, 2, 3, 5, 7, and 11. (Note that 1 is not a prime number.)

Product: The answer to a multiplication question.

Proper fraction: A fraction where the numerator is smaller than the denominator; for example:

$\frac{7}{9}$, $\frac{1}{4}$, $\frac{5}{8}$ and $\frac{13}{15}$ are proper fractions.

Quotient: The answer to a division question.

Reciprocal: A reciprocal is two numbers that when multiplied together equal one; in a fraction, the reciprocal is calculated by multiplying the fraction by its inverse. The numerator and denominator of a fraction are interchanged.

Sum: The answer to an addition question.

Terms: In a fraction, the numerator and denominator:

$$\frac{\text{numerator}}{\text{denominator}} \qquad \frac{3}{8}$$

Unlike fractions: Fractions that do not have the same denominator. For example, ¾ and ⅙ are unlike fractions.

Numbers

Words are formed with letters. Numbers are formed with digits. The ten digits of our number system are:

0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

Place value

By stringing these ten digits together we can form any number. The number system we use is called the *decimal system*, which is based on units of ten. The position that a digit occupies in a number tells us how many ones, tens, hundreds, thousands (and so on) that the number contains. The place values or position values in a number are shown in Figure 1. Each value is ten times greater than the place to its right.

Whole numbers

Whole numbers are complete units without fractional parts. They are the numbers to the left of the decimal point. Numbers larger than zero are called *positive* (+) numbers, whereas numbers that are less than zero are called negative (–) numbers. Zero is neither positive nor negative.

Decimal fractions

If the number has a decimal point, then the position of the digits after the decimal point indicates how many tenths, hundredths, thousandths (and so on) the number contains.

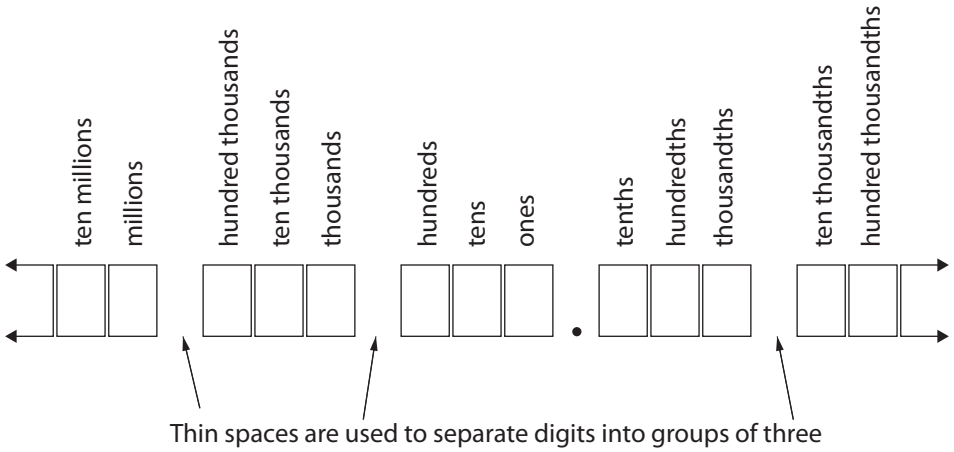


Figure 1 — Place values

Example

The number 23 805.90 represents a number that has the following groups in expanded form:

$$23\,805.90 = 20\,000 + 3000 + 800 + 00 + 5 + \frac{9}{10} + \frac{0}{100}$$

Notice the zero in the tens place is used as a placeholder indicating there are no quantities of ten in the number.

The precision of a measurement describes the smallest unit to which the measurement was made. For example, the measurement 0.0316 cm has a precision of 0.0001 cm or one ten-thousandth of a centimetre. It is important to realize that the measurements 0.6 cm, 0.60 cm and 0.600 cm do *not* mean the same thing. The precision of these three numbers varies from one-tenth to one-thousandth. In this case, the zeros to the right of the number 6 in 0.60 cm and 0.600 cm are significant because they represent units that have actually been counted.

The degree of precision with which a measurement is to be made sometimes determines what measuring instruments will be used. For example, you would have to use a micrometer rather than a steel ruler to measure to a precision of 0.01 mm.

Rounding numbers

The precision required from numbers varies from trade to trade. A carpenter may need a measurement precise to a tenth, while a mechanic may need precision up to a thousandth or more. Rounding is a process that changes a more precise number to a less precise, approximate number that is still adequate for a given situation.

Rounding rules

Rounding is easy if you follow a few simple rules:

- Only round answers after the calculations are complete.

- Decide on the degree of accuracy required.
- Only consider the number to the left of the place value to the right of desired accuracy.
- If the next digit to the right is a 5, 6, 7, 8, or 9, then add 1 to the digit.
- If the next digit to the right is a 0, 1, 2, 3, or 4, then do not change the digit.
- Change all the digits after the place value digit to zeros. If any of the digits after the digit follow a decimal point, then drop them from the number.

Example 1
The estimated population of Canada in 2015 is 35 749 600 people. If you only required an accuracy to the closest million people, then you would round to the nearest million.

Solution
Step 1: Underline the millions digit: 3<u>5</u> 749 600

Step 2: Since the next digit is a 7, the underlined number is rounded up to a 6.

Step 3: Change all the digits after the underlined digit to zeros. Then remove the underline. The population of Canada is approximately 36 000 000.

Adding or subtracting whole numbers or decimal fractions

When adding or subtracting whole numbers or decimal fractions, it is important to add only tenths digits to tenths digits, ones digits to ones digits, tens digits to tens digits, hundreds digits to hundreds digits, and so on. To accomplish this, always make sure that the decimal points are lined up in a column and each decimal fraction has the same number of digits after the decimal point.

Example 1
Add the following:

1.56 + 38 + 0.009 + 0.7

Solution
Arrange the numbers as shown. Note that a decimal point is included in the number 38 and every number has been written with three digits after the decimal point to help align them.

$$\begin{array}{r} 1.560 \\ 38.000 \\ 0.009 \\ 0.700 \\ \hline 40.269 \end{array}$$

Note that the numbers you are adding range in precision from whole numbers to thousandths. If you are doing imprecise work, you would probably have to round the above answer to 40, since one of your original numbers given (38) was only precise to a whole number.

Example 2
Subtract the following: 54.00 – 9.89.

Solution
Align the decimal points. Notice that you must borrow from the 4 and from the 5:

$$\begin{array}{r} {\scriptstyle 13} \\ {\scriptstyle 4\ \cancel{5}\ 9\ 10} \\ \cancel{5}\cancel{4}.\cancel{0}0 \\ -\ 9.89 \\ \hline 44.11 \end{array}$$

When numbers are added, the answer is called a *sum*. The sum of 8 and 5 is 13. When numbers are subtracted, the answer is called a *difference*. The difference of 10 and 4 is 6.

Multiplying whole numbers and decimal fractions

When two numbers are multiplied together, the answer is called a *product*. The numbers being multiplied together are called *factors*. For example, in the equation $2 \times 3 = 6$, the numbers 2 and 3 are factors, and 6 is the product.

When multiplying numbers greater than 9, you must keep the placement of that answer in the same column; for example, tens, hundreds. It is easiest to use a 0 for a placeholder in the lower place value.

Example 1
Find the product of 537 and 274.

Solution
Multiply 537 first by 4, then by 7, and then by 2 as shown. Add these products.

$$\begin{array}{r} 537 \\ \times 274 \\ \hline 2148 \\ 37590 \\ 107400 \\ \hline 147138 \end{array}$$

When multiplying decimal fractions, great care must be taken in placing the decimal point in the product. Counting from the last digit in your answer, put the decimal point as many places to the left as there are total decimal places in the numbers multiplied (see Example 2 below).

Example 2
Multiply 1.35 by 0.409.

Solution
As above, multiply 1.35 first by 9, then by 0 and then by 4. Arrange these products as shown. Since the factors have a total of five decimal places, the product must also have five decimal places.

1.35	2 decimal places
$\times$0.409	3 decimal places
1215	
0000	
54000	
0.55215	5 decimal places

Dividing whole numbers and decimal fractions

When one number is divided by another, the answer is called a *quotient*. The number being divided is called a *dividend*, and the dividing number is called a *divisor*. There is one number that can never be divided into another: zero. It is impossible to divide zero into any number. For example, 5 ÷ 0 is impossible. We say that division by zero is *undefined*.

There are three ways of indicating division:

1. $\text{divisor}\overline{)\text{dividend}}^{\text{quotient}}$

2. dividend ÷ divisor = quotient

3. $\dfrac{\text{dividend}}{\text{divisor}} = \text{quotient}$

For example, the problem of dividing 10 by 2 can be written as:

$$2\overline{)10} \text{ or } 10 \div 2 \text{ or } \frac{10}{2}$$

We read $2\overline{)10}$ as "2 into 10," 10 ÷ 2 as "10 divided by 2," and $\frac{10}{2}$ as "10 over 2."

Example 1
Find 860 ÷ 12 using long division.

Solution
Write the question as 12 into 860. Then work your way from left to right on the dividend, checking to see how many whole times the divisor will go into that portion of the dividend.

a. 12 does not go into 8, so move to the next number; 12 goes into 86 7 times. Place 7 above the 6. $7 \times 12 = 84$. Place 84 below 86 and subtract. Then write the 2 below.

b. Bring down the next number in 860, which happens to be 0, to give you 20 on that line. Now 12 goes into 20 once. Place the 1 above the 0, and $1 \times 12 = 12$. Place 12 below 20 and subtract. Since there is nothing to bring down, 8 is the remainder. Write the quotient as 71 R8.

```
     71
12)860
   84
    20
    12
     8
```

$860 \div 12 = 71$ R8

Whenever whole numbers are divided there is often a remainder. Use the letter *R* to indicate the remainder.

When dividing decimal fractions, take special care in dealing with the decimal points. You can make the task simpler by rearranging the question so that you always divide by a whole number.

Example 2
Find $9.407 \div 0.23$ by long division.

Solution
Write as 0.23 into 9.407. Multiply both the divisor and dividend by 100 and rewrite as 23 into 940.7. This allows you to divide by a whole number. Place the decimal point in the quotient directly above the decimal point in the dividend. Divide as in Example 1. Notice that 23 goes into 20 zero times.

```
0.23)9.407

     40.9
23)940.7
   92
    20
     0
    207
    207
      0
```

$9.407 \div 0.23 = 40.9$

The process of long division can sometimes produce endless non-zero remainders. You can deal with this by rounding the quotient to an appropriate place value.

Example 3
Find 13 ÷ 0.7 by long division. Round the answer to the nearest hundredth.

Solution
Write as 0.7 into 13. Multiply both the divisor and dividend by 10. Rewrite as 7 into 130. Now you can divide by a whole number. Since you must round the quotient to two places after the decimal point, write three zeros after the 130. Divide as before.

$$0.7\overline{)13}$$

$$7\overline{)130}$$

$$\begin{array}{r} 18.571 \\ 7\overline{)130.000} \\ \underline{7} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 3 \end{array}$$

Rounding the quotient, 13 ÷ 0.7 = 18.57

Order of combined operations

Consider the following calculation:

$2 + 3 \times 4$

Is it $5 \times 4 = 20$ or $2 + 12 = 14$? In other words, do you add first or multiply first? Mathematicians have decided upon the following order of operations:

1. Brackets, such as ( ), { } or [ ]
   First, look for brackets and perform all operations within the brackets.

2. Exponents
   Second, perform any exponential operations (we will cover this in a later section).

3. Multiplying and dividing
   Third, perform all multiplying and dividing going from left to right.

4. Adding and subtracting
   The last operations to be performed are adding and subtracting, from left to right.

An easier way to remember the order of operations is to use the acronym BEDMAS. The letters stand for:

Brackets or parentheses	
Exponents	
Division	left to right
Multiplication	left to right
Addition	left to right
Subtraction	left to right

This is an internationally agreed-upon system for dealing with multiple operations.

You can now answer the above question: $2 + 3 \times 4 = 2 + 12 = 14$

Example 1
Find $50 - 2(3 + 8)$.

Solution

1. Add $3 + 8$ inside the brackets.

   $50 - 2(3 + 8) = 50 - 2(11)$

2. Next, multiply $2(11)$. This expression means 2×11. The operation of multiplication is always indicated when a number is placed in front of brackets.

   $50 - 2(11) = 50 - 22$

3. Last, subtract $50 - 22$.

   $50 - 22 = 28$

When working toward a solution, it is safest to perform just one operation at a time.

Example 2
Calculate $7 + 3 \div 0.5 \times 10$.

Solution
First do the division, then the multiplication, and finally the adding.

$7 + 3 \div 0.5 \times 10$

$= 7 + 6 \times 10$

$= 7 + 60$

$= 67$

Example 3
Calculate $2[1.4 \div 0.7 \times 0.5 \div (0.05 + 0.2)]$

Solution
First, do the innermost brackets. Then, within the brackets do multiplying or dividing going from left to right. Finally, multiply by 2.

$2[1.4 \div 0.7 \times 0.5 \div (0.05 + 0.2)]$

$= 2[1.4 \div 0.7 \times 0.5 \div (0.25)]$

$= 2[2 \times 0.5 \div (0.25)]$

$= 2[1.0 \div (0.25)]$

$= 2[4]$

$= 8$

Fractions

Common fractions are numbers that express parts of a whole. Common fractions are written as two numbers, separated by a forward slash or a horizontal line. For example, suppose a pie is cut into five equal pieces and we eat two of them. Then we can say that we ate two-fifths, or ⅖, of the pie.

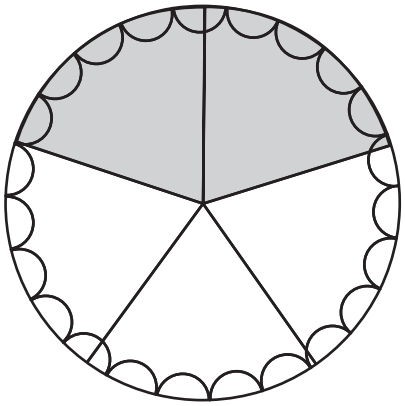


Figure 2 — ⅖ of the pie

Measurements you are required to make on the job seldom turn out to be convenient whole numbers. Instead, they are often fractional amounts. Fractions are used extensively in all trades. In the construction trades many of the materials are still most commonly referred to by their imperial fractional widths. Mechanics encounter fractions in connection with the revolutions of rear axles, crankshafts, ring gears and wheels, in the measurement of parts, and in the adjustment of components of all kinds.

Operations with fractions

A common fraction is always made of two numbers. The top number is called the *numerator* and the bottom number is called the *denominator*. The slash or horizontal line means the same thing as the ÷ sign. So any fraction can also be written out as the numerator divided by the denominator.

$\frac{2}{5}$ ← numerator
← denominator

When working with common fractions it is important to remember that the whole is the same as the number one. If we ate the whole pie, it is the same as eating 5/5 of the pie. Fractions can be used to represent numbers less than one, more than one, or equal to one.

Proper fractions are less than one. Their numerators are less than their denominators.

For example:

$$\frac{1}{2} < 1, \frac{3}{4} < 1, \frac{9}{10} < 1, \text{and } \frac{1}{40} < 1$$

Improper fractions are equal to or greater than one. For example:

$$\frac{4}{3} > 1, \frac{10}{10} = 1, \frac{9}{4} > 1, \text{and } \frac{13}{12} < 1$$

Mixed numbers are composed of a whole number and a proper fraction. Mixed numbers are always greater than one.

For example:

$$1\frac{1}{4} > 1, 5\frac{3}{8} > 1, 10\frac{1}{2} > 1, \text{and } 2\frac{4}{7} > 1$$

The rectangles shown below have been divided into four equal parts. The shaded part can be described by the improper fraction as $\frac{9}{4}$ or by the mixed number $2\frac{1}{4}$.

Example 1

Write $\frac{13}{5}$ as a mixed number.

Solution

The idea is to find the number of $\frac{5}{5}$ in $\frac{13}{5}$ or

$$\frac{13}{5}=\frac{5}{5}+\frac{5}{5}+\frac{3}{5}=1+1+\frac{3}{5}=2\frac{3}{5}$$

The quickest way to write an improper fraction as a mixed number is to divide the numerator by the denominator:

$$\frac{13}{5}\Rightarrow 5\overline{)13}\Rightarrow 2\frac{3}{5}$$
$$\begin{array}{r} 2 \\ 5\overline{)13} \\ \underline{10} \\ 3 \end{array}$$

Example 2

Write $4\frac{3}{8}$ as an improper fraction.

Solution
The idea is to find the number of eighths in $4\frac{3}{8}$:

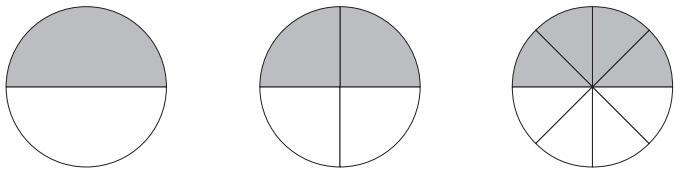
$$4\frac{3}{8}=1+1+1+1+\frac{3}{8}=\frac{8}{8}+\frac{8}{8}+\frac{8}{8}+\frac{8}{8}+\frac{3}{8}=\frac{35}{8}$$

The quickest way to change a mixed number to an improper fraction is to multiply the whole number by the denominator and add this product to the numerator:

$$4\frac{3}{8}=\frac{4\times 8+3}{8}=\frac{32+3}{8}=\frac{35}{8}$$

Equivalent fractions

In each of the following, one-half of the pie is shaded.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$

The common fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are called *equivalent fractions* because they represent the same value. Here, $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$.

There are many—actually infinitely many—ways of expressing the value $\frac{1}{2}$.

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}$$, and so on.

For ease of understanding, it is always best to express fractions in the lowest terms.

Fractions in the lowest terms

A fraction is said to be in the lowest terms when neither the numerator nor the denominator is divisible by a common prime number. For example, $\frac{10}{15}$ is not in the lowest terms, since both 10 and 15 are divisible by 5.

Prime numbers are divisible only by one or themselves:

2, 3, 5, 7, 11, 13, 17, 19, 23, ...

Example 1
Write 140 as a product of primes.

Solution
The process is to keep dividing by the prime numbers 2, 3, 5, 7, 11, ...

Divide 140 by 2.

$$140 = 2 \times 70$$

Divide 70 by 2.

$$140 = 2 \times 2 \times 35$$

35 is not divisible by 2 or 3, so divide 35 by 5.

$$140 = 2 \times 2 \times 5 \times 7$$

The quotient 7 is a prime, so you are done.

The method used in Example 1 is not the only way to write 140 as a product of primes. You could easily have obtained the same answer as follows:

$$140 = 14 \times 10 = 2 \times 7 \times 2 \times 5 = 2 \times 2 \times 5 \times 7$$

Factoring the numerator and denominator of a fraction is one way of helping to reduce the fraction to lowest terms. Prime numbers that occur in both the numerator and denominator can then be cancelled and replaced with a one. For example:

$$\frac{15}{20}=\frac{3\times5}{2\times2\times5}=\frac{3\times\cancel{5}}{2\times2\times\cancel{5}}=\frac{3}{2\times2}=\frac{3}{4}$$

Example 2
Reduce:

a. $\frac{30}{63}$ b. $\frac{66}{24}$ c. $5\frac{6}{42}$

Solution
Use the method in Example 1 to factor the numerator and denominator. Replace crossed out factors with ones, to represent 1/1, equivalent to the whole number 1. Then multiply the remaining factors.

a. $\frac{30}{63}=\frac{2\times3\times5}{3\times3\times7}=\frac{2\times\cancel{3}^{1}\times5}{\cancel{3}_{1}\times3\times7}=\frac{10}{21}$

b. $\frac{66}{24}=\frac{2\times3\times11}{2\times2\times2\times3}=\frac{\cancel{2}^{1}\times\cancel{3}^{1}\times11}{\cancel{2}_{1}\times2\times2\times\cancel{3}_{1}}=\frac{11}{4}$

c. $5\frac{6}{42}=5\frac{2\times3}{2\times3\times7}=5\frac{\cancel{2}^{1}\times\cancel{3}^{1}}{\cancel{2}_{1}\times\cancel{3}_{1}\times7}=5\frac{1}{7}$

There are many shortcuts to reducing fractions, and you may already be aware of some. If you have a method that works, keep using it. Check with your instructor on the validity of your method.

Typically the most common fractions used on a job site are for imperial measurements. The standard imperial tape measure has the whole inch broken into divisions of halves, fourths, eighths, sixteenths, and sometimes thirty-seconds.

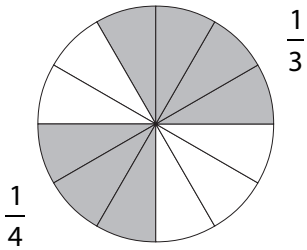
Adding and subtracting common fractions

Notice how you can add the shaded parts of the whole.

$\frac{2}{8}$ $\frac{3}{8}$

$$\begin{array}{r} \frac{2}{8} \\ +\ \frac{3}{8} \\ \hline \frac{5}{8} \end{array}$$

The fractions $\frac{2}{8}$ and $\frac{3}{8}$ have a common denominator. Before adding or subtracting fractions, the fractions must be written with a common denominator. A visual example of where adding fractions that do not have a common denominator may be used, for example $\frac{1}{3}$ and $\frac{1}{4}$, is shown below.



$$\begin{array}{r} \frac{1}{3}=\frac{1\times4}{3\times4}=\frac{4}{12} \\ +\ \frac{1}{4}=\frac{1\times3}{4\times3}=\frac{3}{12} \\ \hline \frac{7}{12} \end{array}$$

Notice that the method for writing $\frac{1}{3}$ as $\frac{4}{12}$ is the reverse method of reducing fractions to lowest terms. As you can see, the quickest way to find a common denominator is to multiply the denominators of the two fractions and reduce if necessary.

Adding like fractions

- Adding like fractions is straightforward. Simply add the numerators and place the sum over the common denominator.

  $$\frac{2}{3}+\frac{2}{3}=\frac{4}{3}=1\frac{1}{3}$$

Adding unlike fractions

- Before you can add unlike fractions, you have to find the lowest common denominator (LCD). Look at the denominators and determine the lowest number that the denominators can divide into evenly. For 2/3 and 1/4 the LCD is 12.

  $$\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}$$

- You then have to express the fractions in higher terms using 12 as the denominator. For 2/3 multiply both the denominator and numerator by 4. For 1/4 multiply both the denominator and numerator by 3.

  $$\frac{1}{4}=\frac{1\times3}{4\times3}=\frac{3}{12}$$

- Then add the numerators and place the sum over the common denominator 12.

  $$\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$$

Adding mixed numbers

- To add mixed numbers, first find the LCD of the fractions and change the fractions to their equivalents, using the LCD for the denominators.
- Then add the numbers. If the sum of the fractions is an improper fraction, you will have to change it to a mixed number and "carry" the whole number.

$$\begin{array}{r} 2\frac{1}{4}=2\frac{4}{16} \\ +3\frac{15}{16}=3\frac{15}{16} \\ \hline 5\frac{19}{16}=6\frac{3}{16} \end{array}$$

Subtracting like fractions

- To subtract like fractions, subtract only the numerators.

$$\frac{7}{8}-\frac{3}{8}=\frac{4}{8}=\frac{1}{2}$$

Subtracting unlike fractions

- To subtract unlike fractions, find the LCD and then subtract the numerators.

$$\frac{3}{8}-\frac{1}{3}=\frac{9}{24}-\frac{8}{24}=\frac{1}{24}$$

Subtracting mixed numbers

- To subtract mixed numbers, first find common denominators and then subtract the numerators of the fraction and then the whole numbers.
- However, when the fractional component of the mixed number being subtracted is greater than the other, you have to rename the whole number.
- Take 1 from the whole number and add to the fraction. In the example 6 8/16, change the 6 to 5 16/16 and add in the fraction 8/16 to get 5 24/16. Then subtract as usual.

$$\begin{array}{r} 6\frac{8}{16}=5\frac{24}{16} \\ -4\frac{15}{16}=4\frac{15}{16} \\ \hline 1\frac{9}{16} \end{array}$$

Example 1

Add $\frac{5}{6}+\frac{3}{8}$.

Solution

First find a common denominator. Think as follows: The larger denominator is 8. Will 6 divide evenly into 8? No. Now double the 8. $8 \times 2 = 16$. Will 6 divide evenly into 16? No. Triple the 8. $8 \times 3 = 24$. Will 6 divide evenly into 24? Yes. The common denominator is 24.

Multiply $\frac{5}{6} \times \frac{4}{4}$ and $\frac{3}{8} \times \frac{3}{3}$ to get equivalent fractions. Rewrite the answer as a mixed number.

$$\begin{array}{r} \frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24} \\ + \frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24} \\ \hline \frac{29}{24} \end{array}$$

$$\frac{29}{24} = 1\frac{5}{24}$$

Example 2

Subtract $9\frac{4}{5} - 6\frac{3}{10}$.

Solution

The common denominator is 10. Subtract the fractions and the whole numbers. Reduce the answer to the lowest terms.

$$\begin{array}{r} 9\frac{4}{5} = 9\frac{4 \times 2}{5 \times 2} = 9\frac{8}{10} \\ - 6\frac{3}{10} = 6\frac{3}{10} \quad = 6\frac{3}{10} \\ \hline 3\frac{5}{10} \end{array}$$

$$3\frac{5}{10} = 3\frac{1}{2}$$

Example 3

Subtract $4\frac{1}{3} - 1\frac{1}{2}$

Solution

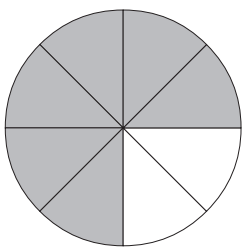
The common denominator is 6. Notice that you cannot subtract $\frac{3}{6}$ from $\frac{2}{6}$. You must borrow a whole $\frac{6}{6}$ from the 4. Then add this $\frac{6}{6}$ to $\frac{2}{6}$. Now you can subtract.

$$\begin{array}{r} 4\frac{1}{3} = 4\frac{1\times 2}{3\times 2} = 4\frac{2}{6} \\ -1\frac{1}{2} = 1\frac{1\times 3}{2\times 3} = 1\frac{3}{6} \\ \hline 4\frac{2}{6} = 3\frac{6}{6}+\frac{2}{6} = 3\frac{8}{6} \\ -1\frac{3}{6} \qquad\qquad = 1\frac{3}{6} \\ \hline 2\frac{5}{6} \end{array}$$

Multiplying common fractions

The pie below is cut into 8 parts. $\frac{3}{4}$ of the pie is shaded. Suppose you want to find $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{1}{2}\times\frac{3}{4}$. Notice that the word *of* means *times* in arithmetic.

$\frac{3}{4}$



$$\frac{1}{2}\text{ of }\frac{3}{4} = \frac{1}{2}\times\frac{3}{4} = \frac{3}{8}$$

When multiplying fractions, multiply the numerator times the numerator and the denominator times the denominator.

Before you multiply fractions, it is sometimes possible to "cancel" by dividing the numerator of one fraction and the denominator of the other by the same number.

$$\frac{\cancel{3}^{1}}{4}\times\frac{1}{\cancel{6}_{2}} = \frac{1}{8}$$

Multiplying mixed numbers

- To multiply a whole number by a fraction, assign 1 as the denominator for the whole number. In this case, give 5 a denominator of 1.

  $$\frac{1}{2}\times\frac{5}{1}$$

- Then multiply as usual.
- To multiply mixed numbers, first change the mixed numbers to improper fractions.

$$\frac{1}{2}\times\frac{5}{1}=\frac{5}{2}=2\frac{1}{2}$$

- Cancel if possible, multiply the numerators, and then multiply the denominators as usual.

$$1\frac{1}{5}\times 2\frac{2}{3}=\frac{\cancel{6}^{2}}{5}\times\frac{8}{\cancel{3}_{1}}=\frac{16}{5}$$

$$=3\frac{1}{5}$$

The steps to multiplying fractions

Step 1: Rewrite any mixed numbers as improper fractions.

Step 2: Factor, if possible, all numerators and denominators.

Step 3: Cancel factors common to both the numerator and denominator.

Step 4: Multiply the remaining factors.

Example 1

Multiply $\frac{2}{3}\times\frac{4}{5}$.

Solution
Simply multiply numerators and denominators:

$$\frac{2}{3}\times\frac{4}{5}=\frac{2\times 4}{3\times 5}=\frac{8}{15}$$

Example 2

Multiply $2\frac{3}{4}\times 5$.

Solution
Write both numbers as improper fractions, then multiply numerators and denominators. Rewrite the answer as a mixed number:

$$2\frac{3}{4}\times 5=\frac{11}{4}\times\frac{5}{1}=\frac{11\times 5}{4\times 1}=\frac{55}{4}=13\frac{3}{4}$$

Example 3

Multiply $\frac{4}{21}\times 3\frac{1}{2}$.

Solution

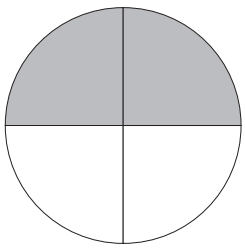
Write $3\frac{1}{2}$ as $\frac{7}{2}$. Factor all numerators and denominators. Cancel common factors. (If you do not cancel first, then you will have $\frac{4}{21}\times\frac{7}{2}=\frac{28}{42}$ and you will have to reduce $\frac{28}{42}$ to lowest terms regardless.)

$$\frac{4}{21}\times 3\frac{1}{2}=\frac{4}{21}\times\frac{7}{2}=\frac{2\times 2\times 7}{3\times 7\times 2}=\frac{\cancel{2}^{1}\times 2\times \cancel{7}^{1}}{3\times \cancel{7}_{1}\times \cancel{2}_{1}}=\frac{2}{3}$$

Dividing common fractions

How many quarters are there in one half? The drawing suggests that there are two quarters in the shaded half of the pie. Mathematically, we are asking, what is one-half divided by one-quarter, or

$\frac{1}{2}$



$$\frac{1}{2}\div\frac{1}{4}=2$$

Notice also that $\frac{1}{2}\times\frac{4}{1}=2$. The number 4 is the *reciprocal* of $\frac{1}{4}$. The reciprocal of a fraction is found by interchanging the numerator and the denominator. Note that your calculator has a reciprocal key. $\boxed{\frac{1}{x}}$

Division questions can be rewritten as multiplication questions. Dividing by a number is the same as multiplying by the reciprocal of that number.

Mixed numbers

- To divide mixed numbers, change them to improper fractions:

  $$1\frac{3}{8}\div\frac{15}{16}=\frac{11}{8}\div\frac{15}{16}=$$

- Find the reciprocal of the second fraction.

- Cancel if possible, multiply the numerators and then the denominators:

  $$\frac{11}{\cancel{8}_{1}}\times\frac{\cancel{16}^{2}}{15}=\frac{22}{15}=1\frac{7}{15}$$

Example 1

Divide $\frac{1}{2} \div \frac{2}{3}$.

Solution
Rewrite as a multiplication question by changing the "÷" to a "×" and the $\frac{2}{3}$ to its reciprocal $\frac{3}{2}$, then multiply:

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{1 \times 3}{2 \times 2} = \frac{3}{4}$$

Example 2

Divide $6\frac{3}{8} \div 5$.

Solution
Rewrite as improper fractions. Then rewrite as a multiplication question. Then multiply and rewrite the answer as a mixed number:

$$6\frac{3}{8} \div 5 = \frac{51}{8} \div \frac{5}{1} = \frac{51}{8} \times \frac{1}{5} = \frac{51 \times 1}{8 \times 5} = \frac{51}{40} = 1\frac{11}{40}$$

Solving word problems

These four steps will help you translate a word problem into a number sentence.

Step 1: What do you have to find out?
Usually, the last sentence in a word problem asks a question. Read this question carefully to be sure you understand what you are being asked to find.

Step 2: What are you told?
Go through the problem, making note of what numbers you are given.

Step 3: What must you do to find the answer?
Key words used in the problem often tell you what mathematical operation(s) you will use to solve the problem. Watch for these words:

- to indicate addition: total, sum, altogether, combined, gained, joined together, increase
- to indicate subtraction: difference, how much more, larger than, how much less, smaller than, lost, leaves how many, shortest, decrease

The way the information is given in the first sentence of multiplication and division problems will usually indicate what operation to use. Here are some examples:

- indicating multiplication:
  - 9 units costing $5.00 each

  - 47 boxes containing 165 items each
  - 215 units at 75 mm each
- indicating division:
  - kilometres per hour
  - cost per item
  - production per day

Step 4: What is your number sentence?
Write a number sentence using the information you've gathered by following the first three steps.

The following example demonstrates how to use the four steps to solve a word problem.

A heavy-duty mechanic requires 112 hours to complete a job. The mechanic works 7 hours each day. How many days will be required to complete the job?

Step 1: What do you have to find out?
The number of days needed to complete the job

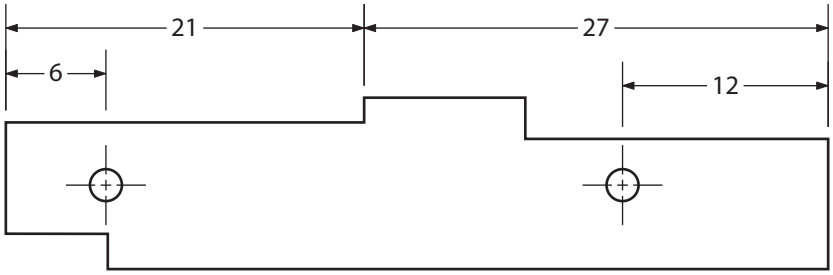
Step 2: What are you told?
7 hours per day; 112 hours for the total job

Step 3 What must you do to find the answer?
Divide 112 hours by 7 to find number of days

Step 4: What is your number sentence?
112 ÷ 7 = 16

Exercise

1. What is the distance between the centres of the 2 holes in the following diagram? All dimensions are in centimetres.



2. A bundle of shingles contains an average of 187 shingles. How many shingles are there in 259 bundles?

3. A welder used 688 rods to weld some brackets. If each bracket required 8 rods, how many brackets were welded?

4. A bin contained 347 hexagonal nuts. Four projects required 18, 26, 14, and 36 nuts respectively. How many nuts were left?

Answers

1. First add 21 and 27 to find total length. Then add 6 + 12 and subtract this sum from total length.

$$\begin{array}{r} 21+27=48 \\ 6+12=\underline{18} \\ 30\text{ cm} \end{array}$$

2. Multiply 187 by 259.

$$\begin{array}{r} 259 \\ \times\underline{187} \\ 1813 \\ 2072 \\ \underline{259} \\ 48433 = 48\,433 \end{array}$$

3. Divide 688 by 8.

$$\begin{array}{r} 86 \\ 8\overline{)688} \\ \underline{64} \\ 48 \end{array}$$

4. First add the numbers of nuts used in the four projects, and then subtract this total from the total number of nuts.

$$\begin{array}{r} 18 \\ 26 \\ 14 \\ +\underline{36} \\ 94 \end{array} \qquad \begin{array}{r} 347 \\ -\underline{94} \\ 253 \end{array}$$

It is always a good idea to check your answer to any word problem by reading or writing a sentence that contains the answer. For example, a written statement designed to accompany question 1 could read: "The distance between the centres of the 2 holes in the diagram is 30 cm."

Converting between common fractions and decimal fractions

It is often convenient to write a common fraction as a decimal fraction. For example, we think of three quarters as 75 cents or $0.75. In other words.

$$\frac{3}{4} = 0.75$$

Convert decimals to fractions

There are three types of decimal fractions:

1. *Terminating decimals* have a fixed number of digits; for example, 0.5, 3.625, 10.128 496.

2. *Repeating decimals* have one digit or a group of digits that repeat forever. A bar is placed over the repeating digit or group of repeating digits; for example:

   $0.333\ 333\ 333... = 0.\overline{3}$

   $6.934\ 934\ 934... = 6.\overline{934}$

   $8.056\ 26262... = 8.05\overline{62}$

3. *Non-repeating decimals* have an infinite number of digits but no repeating digit or group of digits; for example:

   0.123 456 789 101 112...
   6.505 505 550 555 505...
   3.141 592 748 605 213...

Terminating decimals can be written as accurate common fractions. Repeating and non-repeating decimals must be rounded to be written as common fractions. When a decimal is written as a fraction, the decimal digits appear in the numerator of the fraction and the denominator has the same number of zeros as there are decimal digits.

Study the following example:

- Convert 0.125 to a fraction.

  0.125

- Count the number of digits after the decimal point (including zeros). Use that number of zeros in the denominator of the common fraction. In this case, 0.125 has 3 digits after the decimal point, so there are 3 zeros in the denominator.

  $$\frac{125}{1000}$$

- Express the fraction in lowest terms.

$$\frac{125}{1000} = \frac{1}{8}$$

Convert fractions to decimals

- Convert 7/12 to a decimal, expressing your answer to the nearest hundredth.
- Place a decimal point after the number that is the numerator. Add one more zero to this numerator than the place value of indicated precision.

$$12\overline{)7.000}$$

- Divide the denominator into the numerator.

$$\begin{array}{r} 0.583 \\ 12\overline{)7.000} \end{array}$$

- Round your answer to the required precision (in this case to the nearest hundredth).

$$\frac{7}{12} = 0.58$$

Examples of converting fractions to decimals

Example 1

Write $\frac{3}{8}$ as a decimal fraction.

Solution

Write $\frac{3}{8}$ as a long division question. Write 3 as 3.00 and add more decimal place zeros as needed.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\frac{3}{8} = 0.375$$

Example 2

Write $5\frac{1}{3}$ as a decimal fraction.

Solution

Use long division to divide the fraction $\frac{1}{3}$. Note that the quotient has repeating threes. Use a bar over the repeating digit. Place the whole number 5 before the decimal, $5.\overline{3}$.

$$\begin{array}{r} .3\overline{3} \\ 3\overline{)1.00} \end{array}$$

$$5\frac{1}{3} = 5.\overline{3}$$

Estimation

Estimation is a useful tool in mathematical problem solving. Estimating gives you an approximate or "ballpark" answer. It is important to be able to make estimates in your head before you use a calculator, to enable you to double check your answer in case you press the wrong key on the calculator. People often estimate differently; there is no one correct way to estimate. It takes practice to get good at choosing numbers that make estimating easier. Here are some estimating strategies:

1. Round to numbers that are easy for you to compute. For example, about how much is 29×14? Some possible estimates are:

   $25 \times 16 = 400$
   $30 \times 10 = 300$
   $30 \times 15 = 450$

2. Use numbers that make sense to you. When you round, sometimes the result will be an underestimate (less than the exact answer) and sometimes it will be an overestimate (more than the exact answer).

problem	estimate	the answer is...
31×4	$30 \times 4 = 120$	a little more than 120
48×5	$50 \times 5 = 250$	a little less than 250
17×21	$20 \times 20 = 400$	about 400

3. Choose numbers that are compatible and easy to work with for the problem. For example, how much is 2775 divided by 6? Some possible estimates are:

   $3000 \div 6 = 500$
   $2800 \div 7 = 400$

4. Use whole numbers to estimate fractions and decimals. For example:

problem	estimate
19.32 – 7.29	19 – 7 = 12
3.7 × 8.2	4 × 8 = 32
$16\frac{2}{3}+12\frac{1}{8}$	17 + 12 = 29
$20\frac{3}{4}\div 3\frac{1}{5}$	21 ÷ 3 = 7

5. Group numbers to make estimation easier. For example, 48 + 27 + 55 + 75 + 98 is approximately = 300. This is because 48 + 55 is about 100, 27 + 75 is about 100, and 98 is about 100.

Using a calculator

The calculator is an excellent tool for saving time and performing more complex calculations. Modern scientific calculators generally have many more features than a standard four- or five-function calculator, and the feature set differs among manufacturers and models. It is therefore important that you check the instruction manual for your calculator to learn to use it properly. Some of the features of a scientific calculator that you will be using in the more advanced Learning Tasks include:

- scientific notation
- floating point arithmetic
- trigonometric functions
- exponential functions and roots beyond the square root
- quick access to constants such as π (pi)

Figure 3 — Scientific calculator

Make yourself familiar with these features. It is also very important that your calculator is in the right mode. For the majority of your work the mode should be in "deg" or degree mode. This notation will appear in the screen of your calculator

Go to the Open University website to see the online unit "Using a scientific calculator."
http://www.open.edu/openlearn/science-maths-technology/mathematics-and-statistics/mathematics-education/using-scientific-calculator/content-section-0

Now complete the Learning Task Self-Test.

Self-Test 1

1. Round 5237.02056 to the nearest thousandth.
   a. 5000
   b. 5237.02
   c. 5237.021
   d. 5237.0206

2. Round 12 465.078 to the nearest thousand.
   a. 12 465
   b. 12 000
   c. 10 000
   d. 12 465.078

3. 13.963 + 335.021 + 2267.123 =
   a. 2616.17
   b. 2616.107
   c. 2616.296
   d. 2616.182

4. Add 2.72, 0.6, 110, and 17.225.
   a. 130.545
   b. 135.945
   c. 136.545
   d. 131.545

5. 11.25 – 3.875 =
   a. 7.15
   b. 7.875
   c. 7.375
   d. 15.125

6. Multiply 19.5 by 7.65.
   a. 14917.5
   b. 14.9175
   c. 1491.75
   d. 149.175

7. $18.753 \div .87 =$
   a. 210.55
   b. 21.555
   c. 2.1555
   d. 0.21555

8. Divide 0.0028 by 0.95. Express your answer to the nearest thousandth.
   a. 0.003
   b. 0.002
   c. 0.0029
   d. 0.00295

9. $178 + 18 \times 6 =$
   a. 286
   b. 295
   c. 1176
   d. 19 224

10. $1890.33 - 543.48 + 101 =$
    a. 1447.52
    b. 2534.81
    c. 2534.85
    d. 1447.85

11. $58 - 33 \times 4 =$
    a. 74
    b. −74
    c. 100
    d. −100

12. Reduce $^{24}/_{64}$ to lowest terms.
    a. $^{12}/_{32}$
    b. $^{6}/_{16}$
    c. $^{3}/_{8}$
    d. $^{1.5}/_{4}$

13. Reduce $^{150}/_{275}$ to lowest terms.
    a. $^{6}/_{11}$
    b. $^{3}/_{5.5}$
    c. $^{3}/_{11}$
    d. $^{12}/_{22}$

14. Change $^{34}/_{8}$ into a mixed number.
    a. $3^{4}/_{8}$
    b. $4^{1}/_{4}$
    c. $4^{1}/_{8}$
    d. $4^{1}/_{16}$

15. Express $5^{7}/_{16}$ as an improper fraction.
    a. $^{103}/_{16}$
    b. $^{57}/_{16}$
    c. $^{87}/_{16}$
    d. $^{35}/_{16}$

16. $^{1}/_{4} + {}^{5}/_{16} + {}^{7}/_{32} =$
    a. $^{7}/_{16}$
    b. $^{13}/_{32}$
    c. $^{25}/_{32}$
    d. $^{13}/_{16}$

17. $^{5}/_{8} + {}^{9}/_{32} - {}^{1}/_{16} =$
    a. $^{7}/_{8}$
    b. $^{7}/_{16}$
    c. $^{27}/_{32}$
    d. $^{13}/_{16}$

18. $\frac{7}{8} \times 5 =$

    a. $\frac{35}{40}$

    b. $4\frac{3}{8}$

    c. $6\frac{3}{8}$

    d. $4\frac{3}{4}$

19. $\frac{3}{16} \times \frac{5}{8} =$

    a. $\frac{9}{32}$

    b. $\frac{9}{16}$

    c. $\frac{80}{24}$

    d. $\frac{15}{128}$

20. $4\frac{1}{2} \times 4 =$

    a. 18

    b. $8\frac{1}{2}$

    c. $16\frac{1}{2}$

    d. $16\frac{1}{8}$

21. $\frac{3}{8} \div \frac{1}{16} =$

    a. 6

    b. 12

    c. $\frac{3}{16}$

    d. $\frac{3}{128}$

22. $\frac{5}{16} \div 4 =$

    a. $\frac{20}{16}$

    b. $1\frac{1}{4}$

    c. $\frac{5}{16}$

    d. $\frac{5}{64}$

23. $\frac{17}{32} + \frac{1}{2} \times \frac{1}{4} =$

    a. $\frac{8}{32}$

    b. $\frac{21}{32}$

    c. $\frac{23}{32}$

    d. $1\frac{9}{32}$

24. The risers for a set of stairs are each 19.4 cm. Each step is 2.5 cm thick. There are 10 risers and 9 steps. What is the height of the stairway?
    a. 216.5 cm
    b. 219 cm
    c. 194 cm
    d. 222 cm

25. A can contains 4.225 L of lacquer. The lacquer is to be divided equally into five containers. How much lacquer will be in each container?
    a. 0.888 L
    b. 0.879 L
    c. 0.845 L
    d. 0.854 L

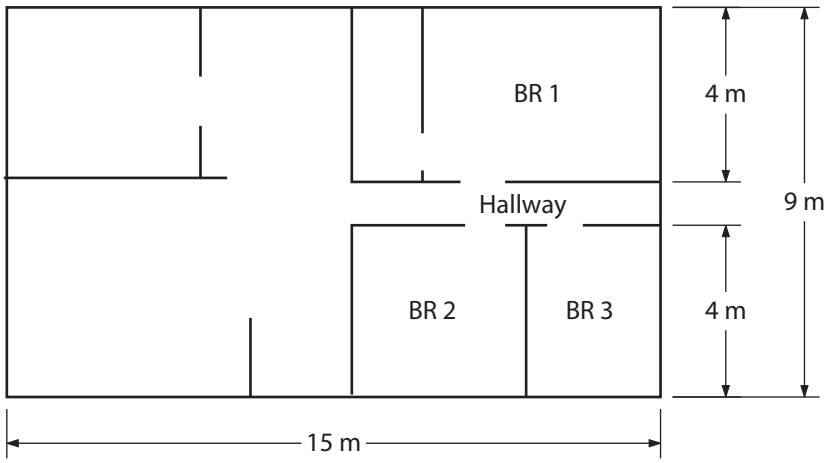
26. A shop used a total of 57 400 screws to assemble an order. If each unit assembled required 41 screws, how many units were completed for the order?
    a. 410
    b. 140
    c. 4100
    d. 1400

27. A coil of wire has 45 layers of wire wound around the core. Each layer has 68 turns of wire. How many turns of wire are there in the coil?
    a. 3060
    b. 3600
    c. 3006
    d. 6030

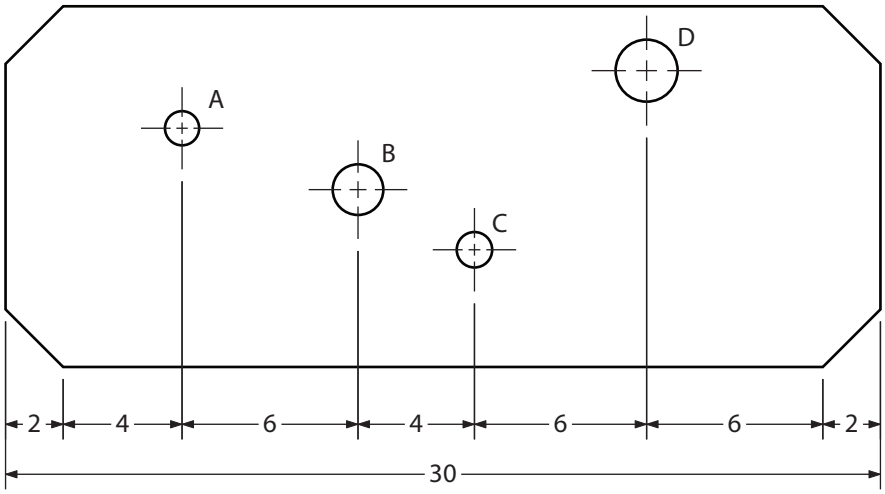
28. If a contractor had 600 m^2 of oak flooring and had two jobs requiring 245 m^2 each, there would be 355 m^2 remaining.
    a. True
    b. False

29. An electrician wires five buildings. The buildings require 42, 65, 73, 69, and 58 outlets respectively. There have been 307 outlets installed.
    a. True
    b. False

30. What is the width of the hallway in the following floor plan?
    a. 2 m
    b. 4 m
    c. 1 m
    d. 3 m



31. On the sheet metal layout below, the distance between the centres of hole A and hole D, measuring lengthwise, is 16 cm. All dimensions are in centimetres.
    a. True
    b. False



32. A cabinetmaker had a bin of 1000 hinges. Six projects required 38, 56, 42, 24, 46, and 34 hinges respectively. How many hinges were left?
    a. 240
    b. 760
    c. 794
    d. 476

33. A total of 256 bronze castings were machined. Each casting had a mass of 793 g, and 158 g were removed from each by machining. The total mass of bronze removed by machining was 162 560 g.
    a. True
    b. False

34. An automotive shop had the following amounts of wire: 84 m black wire; 183 m orange wire; 352 m green wire; 146 m brown wire; 47 m red wire; 316 m blue wire; 329 m white wire. The total length of wire on inventory was 1457 m.
    a. True
    b. False

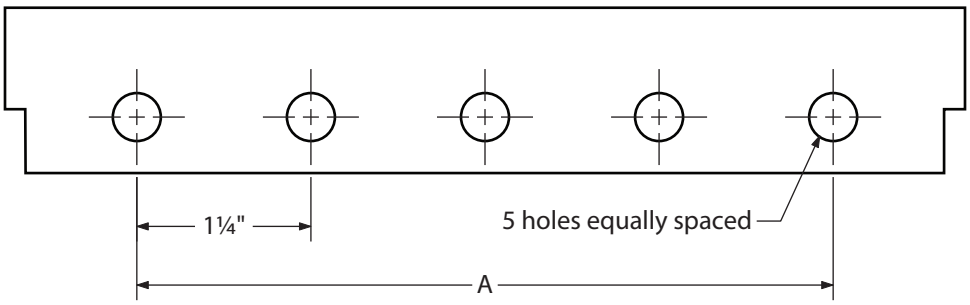
35. An apartment building requires 2475 m of electrical cable. How many 75 m rolls will be needed?
    a. 30
    b. 31
    c. 32
    d. 33

36. A countertop is made of ¾" particleboard and is covered with 1/32" plastic laminate. The width of metal edging required to finish off the edge is 27/32".
    a. True
    b. False

37. In one minute, a large reduction gear turns 6⅓ revolutions. How many revolutions will it turn in 27½ minutes?
    a. 174⅙
    b. 162⅙
    c. 168⅓
    d. 170⅓

38. 12 billets of cold drawn steel 1½" long can be cut from a bar 20" long, if ⅛" is allowed for each saw cut.
    a. True
    b. False

39. Calculate the dimension A in the diagram below
    a. 6 in.
    b. 5 in.
    c. 2½ in.
    d. 6½ in.



40. A 3⁄16" sheet of metal is thicker than a 13⁄64" fastener.
    a. True
    b. False

41. Eight pins, each 4⅜" long, have to be cut from a piece of drill rod. There is a 3⁄32" waste for each cut. What is the minimum length of rod you will need?
    a. 35"
    b. 35¾"
    c. 35 21⁄32"
    d. 35 43⁄64"

42. The diameter of a steel shaft is reduced 7⁄1000". The original diameter was ⅞". The new diameter of the shaft is 0.868".
    a. True
    b. False

43. A good estimation of 1010×21 is

    a. 21 210

    b. 21 000

    c. 20 000

    d. 21 200

LEARNING TASK 2

Work with measurements

To "measure" something means to describe characteristics of that thing with numbers. You can answer questions like, How long is it? How much does it weigh? How much space does it occupy? How hot or cold is it? What is its resistance or voltage?

When you have completed this Learning Task, you should be able to:

- use the common metric units for temperature, length, area, volume/capacity, and mass
- use the common imperial units for temperature, length, area, volume/capacity, and force
- convert between and within metric and imperial units using tables and/or calculators
- take and read measurements with common measuring tools (e.g., thermometer, ruler, measuring tape, triple beam balance, bathroom scale, stop watch, Vernier caliper, micrometer)
- describe and apply precision, accuracy, and tolerance
- estimate in metric and imperial units of measurement

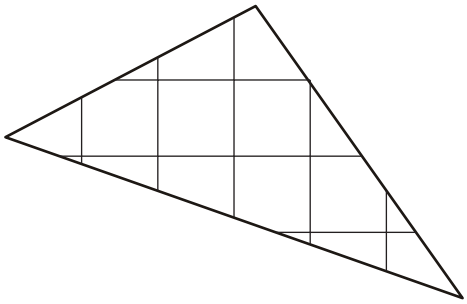
The metric system

In 1960, after long discussions, the world finally agreed on a system of measure, known as the International System of Units or Le Système International d'Unités, or simply SI. SI is a modern version of the old metric system. Over 90% of the world's population is either using or changing to SI. Canada is one of those countries. SI is the simplest system of measurement ever invented, because it, like our number system, is based on the number 10 and units can be expressed as decimal fractions.

In this section, we will study the SI units of temperature, length, area, mass, and volume.

Key terms

Area: The amount of surface enclosed by a figure, measured in square units, such as square centimetres (cm^2).



Area = approximately 8 cm^2

Other metric area units include the square kilometre (km^2), hectare (ha), and square millimetre (mm^2). Imperial area units include the square foot ($ft.^2$) and acre (A).

Celsius: The metric temperature scale, named for Anders Celsius. Sometimes called the *centigrade* scale, because there are 100 Celsius degrees between the freezing point of water (0°C) and the boiling point of water (100°C).

Length: The distance between two points. Metric units of length include the metre (m), centimetre (cm), and kilometre (km). Imperial units of length include the inch (in. or "), foot (ft. or '), and mile (mi.).

Mass: The amount of matter in an object, measured in grams (g), kilograms (kg), or tonnes (t).

Prefixes: The multiplier placed in front of a metric unit. *Kilogram,* for example, means 1000 times larger than one gram. The prefix *kilo* is the multiplier. See Appendix B for a list of metric prefixes.

SI: An abbreviation for the full name of what is usually called the metric system: the International System of Units (or Système International d'Unités).

Volume/capacity: volume is the space taken up by the object itself, while capacity refers to the amount of substance, like a liquid or a gas, that a container can hold.

Metric units of volume include the cubic centimetre (cm^3), and cubic metre (m^3). Imperial volume units include the cubic foot ($ft.^3$). Metric units of capacity include litre (L) The imperial unit of capacity is the gallon (gal.).

Weight: The amount of gravitational force exerted on an object. The metric unit of weight is the newton. Imperial units of weight include the ounce (oz.), pound (lb.), and ton.

Metric prefixes

The most common metric base units and their symbols are shown in the following table.

Quantity	Unit	Symbol
Length	metre	m
Mass	gram	g
Volume	litre	L

You are probably familiar with the terms *kilometre* or km, *kilogram* or kg, and *kilowatt* or kW. In each of these words, the prefix is *kilo*. Kilo always means 1000. Hence, one kilometre is 1000 metres, one kilogram is 1000 grams, and one kilowatt is 1000 watts. Study the following table of metric prefixes and their meanings.

Prefix	Symbol	Meaning (multiply base unit by:)
mega	M	1 000 000
kilo	k	1000
hecto	h	100
deca	da	10
deci	d	0.1
centi	c	0.01
milli	m	0.001
micro	µ	0.000001

There are other prefixes such as giga, G, which means 1 000 000 000 (one billion times). You may have heard the term *gigabytes*, which refers to memory capacity of computers.

The first thing you should notice from the table above is that all the prefixes are based on the number 10. Secondly, note that *mega* and *micro* are most often used in science and technology. Nuclear bombs are measured in megatons of TNT. The biological stuff of cells is measured in micrometres. Thirdly, note that the prefixes *hecto*, *deca*, and *deci* have only limited use in everyday life. The most commonly used prefixes are *kilo*, *centi*, and *milli*. Lastly, do not confuse the symbols M, m, and µ.

Example 1
Convert 3.2 kilometres to metres.

Solution
The question is 3.2 km = _____ m.

The prefix *kilo* means 1000. To convert, multiply 3.2 by 1000 (slide the decimal three places to the right).

$$3.2 \text{ km} = 3.2 \times 1000 = 3200 \text{ m}$$

Example 2
Convert 640 millilitres to litres.

Solution
The question is 640 mL = _____ L.

The prefix *milli* means 0.001. To convert, multiply 640 by 0.001.

$$640 \text{ mL} = 640 \times 0.001 = 0.64 \text{ L}$$

To convert from a smaller unit such as mL to a larger unit, L, we should divide by the conversion factor. Recall that multiplying by 0.001 is the same as dividing by 1000.

For example:

$$432 \div 1000 = 432 \times \frac{1}{1000} = 432 \times 0.001 = 0.432$$

Another way of converting in the metric units is by using a chart like the following:

Kilo 1000×	**Hecto 100×**	**Deca 10×**	**Base Unit**	**Deci 0.1×**	**Centi 0.01×**	**Milli 0.001×**
km kilometre	hm hectometre	dam decametre	m metre	dm decimetre	cm centimetre	mm millimetre
kL	hL	daL	L - litre	dL	cL	mL
kg	hg	dag	g - gram	dg	cg	mg

Example 3
Change 9.82 metres to millimetres.

1. Count the number of places from m to mm (3 places).

2. Move the decimal the same number of places you counted and in the same direction (3 places to the right).

3. Fill in any blank spaces with a zero.

$$9.82 \text{ m} = ? \text{ mm}$$
$$9.82\_ \text{ m} = 9820 \text{ mm}$$

Example 4
Change 5.2 centigrams to grams.

1. Count the number of places from cg to g (2 places).

2. Move the decimal the same number of places you counted and in the same direction (2 places to the left).

3. Fill in any blank spaces with a zero.

5.2 cg = ? g

5.2 cg = .052 g

Length

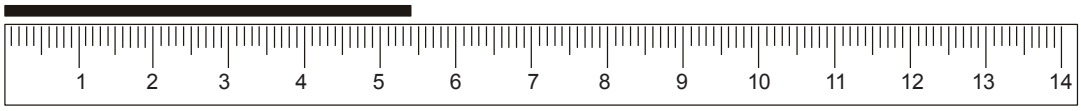
The SI unit of length is the metre.

Notable Lengths	
doorknob height above floor	1 m
thickness of a dime	0.001 m (1 mm)
Olympic sprint distance	100 m
distance from North Pole to Equator	10 000 000 m
size of bacteria	0.000 001 m (1 µm)

Besides the metre, the most commonly used units of measurement are the kilometre (km), the centimetre (cm), and the millimetre (mm).

Look at a metric tape measure and extend it 1 metre. If this distance were laid end to end 1000 times in a straight line, the distance covered would be 1 kilometre. Examine the metre closely; it is divided into 1000 tiny spaces. Each space is 1 millimetre, and there are 10 millimetres in 1 centimetre. Notice that there are 100 centimetres in 1 metre. And, there are 1 000 000 mm in 1 kilometre.

In the illustration below, a bar is placed beside a centimetre ruler. The bar can be measured in both centimetres and millimetres. Its length is 5.4 cm and it is also 54 mm.



Length conversions

Study the chart below.

Unit	Symbol	Relationship
kilometre	km	1 km = 1000 m
metre	m	1 m = 100 cm
centimetre	cm	1 cm = 10 mm
millimetre	mm	1 m = 1000 mm

To convert from one length unit to another, use the familiar rule:

1. When converting a larger unit to a smaller unit, multiply by the conversion factor.

2. When converting a smaller unit to a larger unit, divide by the conversion factor.

Example 1
Convert 2030 metres to kilometres.

Solution
Because you are converting from a smaller unit to a larger unit, divide by 1000, since 1 km = 1000 m.

2030 m = 2030 ÷ 1000 = 2.03 km

Example 2
Sandra's height is 1.63 metres. How many centimetres tall is she?

Solution
Converting from m to cm is converting from a larger to a smaller unit, so we must multiply by 100, since 1 m = 100 cm.

1.63 cm = 1.63 × 100 = 163 cm

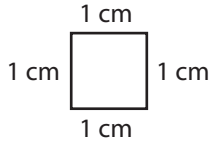
Area

The calculation and application of areas are basic to every trade. Any estimate of the quantity of materials required for a job will be based on calculations of this kind.

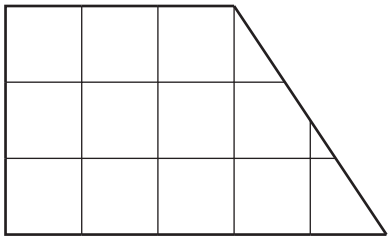
The standard unit of area in SI is the square metre (m^2). The number 2, in the symbol m^2, means that a metre has been multiplied by a metre. It does not mean that anything else has to be multiplied by 2. It should only be taken as a signal that area units are involved in the measurements.

A square metre is the amount of surface contained by a square measuring 1 metre on each side. The surface of a standard door is about 2 m^2. We cannot draw a full square metre on this page; this page is only 0.06 m^2. It would take almost 17 of these pages to cover 1 m^2.

This is a square centimetre (cm^2) shown at approximately actual size:



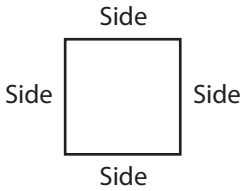
Suppose you want to find the area of the surface outlined below (in cm^2). Your task then is to determine the number of square centimetres that fit into (or cover) the surface.



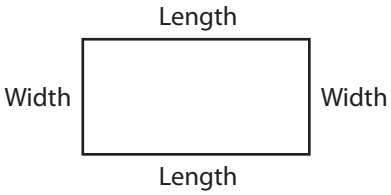
Count the squares (and half squares) and state your answer. _____ cm^2

Did you get 12 or 12.5 cm^2? If you did, good!

Drawing and counting square centimetres on certain surfaces is very tedious. Fortunately there are some very simple formulas you can use to find the areas of squares and rectangles.



Area of square = side × side

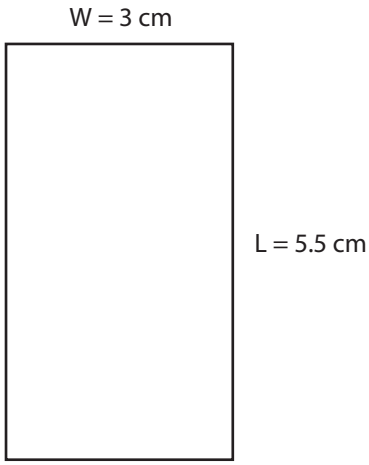


Area of rectangle = length × width

Example
Find the area of the rectangle below.

Solution
Measure the length and width in cm. Use the area formula,

$$A = L \times W$$
$$= 5.5 \text{ cm} \times 3 \text{ cm}$$
$$= 16.5 \text{ cm}^2$$



Notice, again, that cm × cm is written cm^2 to denote the area unit.

Area conversions

The common SI area units are the square kilometre (km^2), the hectare (ha), the square metre (m^2), and the square centimetre (cm^2).

Some notable areas	
area of Canada	10 000 000 km^2
area of small hobby farm (5 acres)	2 ha
area of a door	2 m^2
area of a large postage stamp	5 cm^2

Note that the hectare (ha) is not represented using a superscripted two (2), unlike all other metric area units. Neither does the acre, the imperial area unit, use a superscript in its measure.

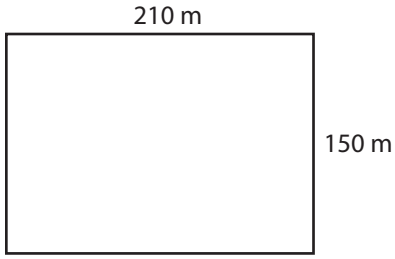
Study the chart below.

Unit	Symbol	Relationship
square kilometre	km^2	1 km^2 = 1 000 000 m^2
hectare	ha	1 km^2 = 100 ha
square metre	m^2	1 ha = 10 000 m^2
square centimetre	cm^2	1 m^2 = 10 000 cm^2

The rule for converting area units is the same as that for converting length units.

Example

A rectangular field measures 210 m by 150 m. How many hectares is this?



Solution

First, find the area of the field in square metres.

$$210 \text{ m} \times 150 \text{ m} = 31\,500 \text{ m}^2$$

To convert the smaller m^2 to the larger ha, divide by the conversion factor, 10 000.

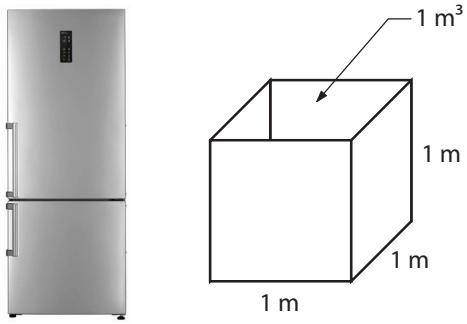
$$31\,500 \text{ m}^2 = 31\,500 \div 10\,000 = 3.15 \text{ ha}$$

Volume and capacity

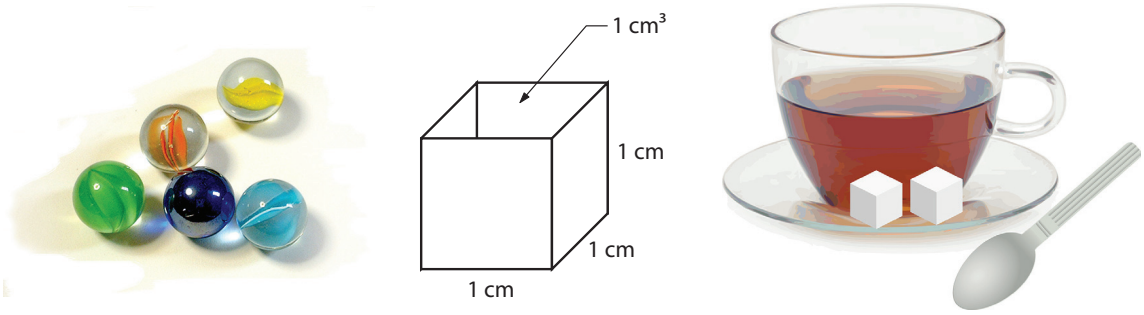
To measure an object's volume is to measure the amount of three-dimensional space it takes up. Common SI units of volume are the cubic metre (m^3), the cubic centimetre (cm^3). The capacity units are kilolitre (kL), the litre (L), and the millilitre (mL).

The cubic metre and cubic centimetre are most often used to measure the volume of solid objects, while kilolitres, litres, and millilitres are used to measure fluid (liquid and gas) volumes. You are probably already used to buying milk and gasoline in litres.

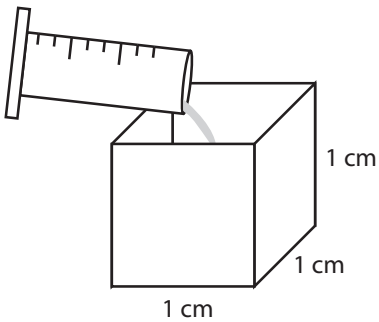
A cubic metre is quite a large amount of space. A cube measuring 1 m on all three sides is exactly 1 m^3. (The expression 1 m^3 is obtained by multiplying 1 metre × 1 metre × 1 metre.) A refrigerator occupies almost 1 m^3.



An ordinary marble has a volume of about 1 cubic centimetre. A sugar cube is about 3 cm^3.

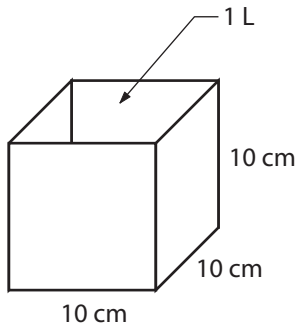


There is a connection between cm^3 and mL. They both measure exactly the same volume! 1 mL of water would exactly fill an empty 1 cm cube.

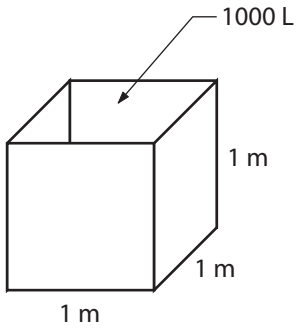


1 mL of water

If we construct a 1000 cm^3 cube, it would measure 10 cm by 10 cm by 10 cm (which would hold 1000 mL of water). The volume of the cube we have constructed holds exactly 1 litre.



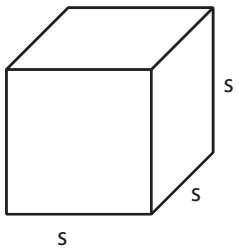
If we construct a cube that measures 1 m by 1 m by 1 m, or 1 m^3, it would hold 1000 litres or 1 kilolitre.



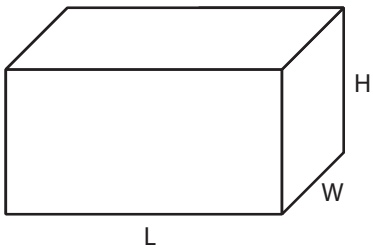
Some volume and capacity relationships are as follows:

Unit	Symbol	Relationship
kilolitre	kL	1 kL = 1000 L
cubic metre	m^3	1 m^3 = 1 kL
litre	L	1 L = 1000 mL
cubic centimetre	cm^3	1 cm^3 = 1 mL
millilitre	mL	1 mL = 1 cm^3

The volume formulas for the cube and rectangular box are as follows.



Volume = side × side × side
V = s × s × s



Volume = Length × Width × Height
V = L × W × H

Example
A box measures 80 cm by 30 cm by 20 cm. How many litres of potting soil can it hold?

Solution
First, find its volume.

$$V = L \times W \times H = 80 \text{ cm} \times 30 \text{ cm} \times 20 \text{ cm} = 48\,000 \text{ cm}^3$$

To convert the smaller unit, cm^3, to the larger unit, L, divide by the conversion factor, 1000.

$$48\,000 \text{ cm}^3 = 48\,000 \div 1000 = 48 \text{ L}$$

Mass

The SI unit of mass is kilogram, kg. Mass and weight are not the same. Mass is the measure of the total substance of an object. Weight is a measure of the gravitational pull on an object. An astronaut might be weightless in outer space, but her mass would be unchanged. The SI unit of weight is the newton. Other metric units of mass are the tonne (t), the gram (g), and the milligram (mg).

Some notable masses	
an elephant	7000 kg or 7 t
a heavyweight boxer	100 kg
1 litre of milk	1 kg
a raisin	0.001 kg or 1 g
a postage stamp	0.02 g or 20 mg

Study the mass conversion table below.

Unit	Symbol	Relationship
tonne	t	1 t = 1000 kg
kilogram	kg	1 kg = 1000 g
gram	g	1 g = 1000 mg
milligram	mg	

Example
A grain of rice weighs 0.25 g. How many grains of rice would be in 1 tonne of rice?

Solution
The problem involves converting 1 tonne to grams and then dividing by 0.25 grams.

1 t = 1000 kg = 1 000 000 g
Now 1 000 000 ÷ 0.25 = 4 000 000

There are 4 000 000 grains of rice in 1 tonne of rice.

Relationship between volume, capacity and mass

To make the following conversions you must know the following equivalents:

Volume		Capacity		Mass (water only)
1 cm^3	=	1 mL	=	1 g
1 dm^3	=	1 L	=	1 kg
1 m^3	=	1 kL	=	1 tonne

With these equivalents you can convert between volume and capacity for all shapes. If the shape is filled with water, you may also convert to mass.

Example 1

You are given the problem: 6000 mL = ? dm^3.

a. Ask yourself, "What am I given, volume, capacity, or mass?"
   In this case you are given mL, a unit of capacity.
b. Ask yourself, "What am I converting to, volume, capacity, or mass?"
   In this case you are asked to convert to dm^3, a unit of volume.
c. Next ask, "What is the equivalent to the given units of capacity in the requested volume?"
   You are given mL and 1 mL (capacity) = 1 cm^3 (volume).
d. Change the given units to this. You are actually changing the name only, not the numbers.
   6000 mL = 6000 cm^3
e. Finally, convert your units to the requested units by sliding your decimal as required.
   6000 cm^3 = 6 dm^3 (decimal moves three places left).

Temperature

The common unit of temperature is the degree Celsius or °C. The Celsius scale was devised with the freezing and boiling point of water in mind.

Some notable temperatures	
Water freezes	0°C
Water boils	100°C
Normal body temperature	37°C
Normal room temperature	20°C
Coldest temperature possible (absolute zero)	–273°C
Surface temperature of the sun	about 5000°C
Core temperature of the sun	about 100 000 000°C

The temperature scale previously used in Canada, which is still used in the United States, is the Fahrenheit scale (F). On the Fahrenheit scale the freezing point of water is 32 degrees Fahrenheit (32°F) and the boiling point is 212 degrees Fahrenheit (212°F). This puts the boiling and freezing points of water exactly 180 degrees apart, whereas on the Celsius scale, the freezing and boiling points of water are 100 degrees apart. You can see that the Celsius unit of measure is larger; a change in temperature of 1 degree Celsius represents a change of 1.8 degrees Fahrenheit.

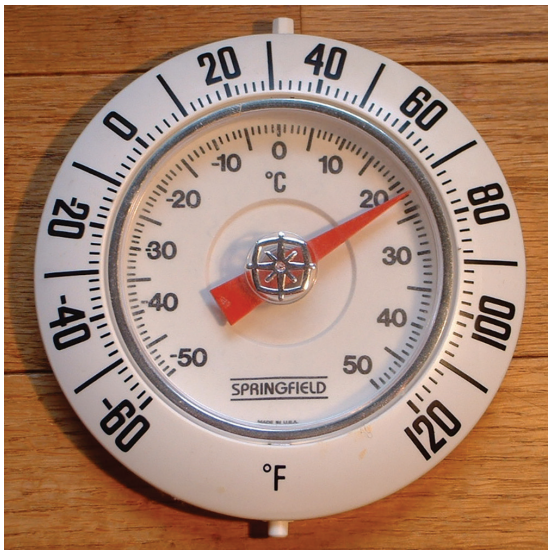


Figure 1 — Thermometer with Fahrenheit and Celsius units

To get an approximate conversion between the two scales, use these formulas:

- Fahrenheit to Celsius: Subtract 32 and halve the resulting number.
- Celsius to Fahrenheit: Double the number and add 32.

For an accurate conversion, use the following formulas:

- Fahrenheit to Celsius: $(°F - 32) \div 1.8 = °C$
- Celsius to Fahrenheit: $(°C \times 1.8) + 32 = °F$

Imperial system of measurement

Canada is not totally metric. When we shop, the meat and vegetable prices are given in per pound as well as per kilogram. We still talk about how tall we are in feet and inches. Recipes still use cups and teaspoons.

The common imperial units of measurement are:

Quantity	Unit	Symbol
length	foot	ft.
volume	gallon	gal.
weight	pound	lb.

The following table contains some of the most common imperial conversions.

Length	Liquid Measure	Weight
1 foot (ft.) = 12 inches (in.)	1 pint (pt.) = 2 cups	1 pound (lb.) = 16 ounces (oz.)
1 yard (yd.) = 3 feet	1 quart (qt.) = 2 pints	1 ton = 2000 lb.
1 mile (mi.) = 5280 feet	1 gallon (gal.) = 4 quarts	

When converting between units in the imperial system, use the familiar rule:

1. When converting a larger unit to a smaller unit, multiply by the conversion factor.
2. When converting a smaller unit to a larger unit, divide by the conversion factor.

Example 1
A two-by-four measures 5 feet 6 inches. How long is it in inches?

Solution
Since 1 foot = 12 inches, the conversion factor is 12, and since you are converting a larger unit to a smaller unit, multiply by 12.

5 feet = 5 × 12 = 60 inches

Now add the 6 inches to the 60 inches.

5 feet 6 inches = 60 + 6 = 66 inches or 5'6" = 66"

Note that "quotes" are sometimes used to represent feet and inches. For example, 8' means 8 ft., and 12" means 12 in..

Example 1
A block wall is 16 rows high and each block is 8" high. How high is the wall in feet and inches?

Solution
Each block is 8" times 16 levels, so multiply 8" times 16 to get the height in inches:

8" × 16 = 128"

Divide the inches by the conversion factor of 12 to get the number of feet.

128" ÷ 12 = 10.667'

Then convert the decimal portion of the foot measurement back into inches. This can be done a couple of ways:

- 0.667' × 12 = 8"

or

- 128" – (10' × 12) = 8"

Therefore, the total height of the wall is 10' 8".

Metric and imperial conversions

Most shops have conversion charts that give the conversions between the metric and imperial systems most useful to your trade. Specification guides will usually give you both metric and imperial measurements if appropriate.

Your calculator may also have the capacity to convert between the two systems. Read your calculator's instructions carefully; ask your instructor if you need help in figuring out how to use this feature.

U.S. measurements of capacity (pints, quarts, gallons) are different from imperial measurements of capacity. For example, a U.S. gallon is 32 ounces, or 3.785 L, while an imperial gallon is 40 ounces, or 4.546 L. There are some differences between U.S. and imperial measurements of mass, as well. Be sure that your conversion calculator or conversion chart uses the same measurement system you are working with.

The chart below gives the equivalents between commonly used metric and imperial measurements. The conversions that are not exact have been rounded off to the nearest thousandth.

Metric/U.S. imperial conversion factors

	Metric to imperial	Imperial to metric
Length	1 km = 0.621 mi. 1 m = 1.094 yd. 1 m = 3.28 ft. 1 cm = 0.394 in. 1 mm = 0.039 in.	1 mi. = 1.61 km 1 yd. = 0.914 m 1 ft. = 0.305 m 1 in. = 2.54 cm 1 in. = 25.4 mm
Mass	1 kg = 2.20 lb. 1 g = 0.035 oz. 1 t (1000 kg) = 2204.6 lb.	1 oz. = 28.4 g 1 lb. = 0.454 kg
Capacity	1 L = 2.11 pt. 1 L = 1.06 qt.	1 pt. = 0.47 L 1 qt. = 0.95 L 1 U.S. gal. = 3.79 L 1 Imp. gal. = 4.55 L

Examples of converting between metric and imperial measurements

It is not necessary to memorize the chart. You will be able to refer to a conversion chart for any calculations.

When you are using an approximate conversion factor, round your answers to the precision of your approximate factor, if necessary.

Converting units of length

Convert 38 mm to inches.

- Multiply mm by the metric to imperial conversion factor for millimetres to inches.

$38 \times 0.039 = 1.482$
38 mm = 1.482 in.

Convert 123 miles to kilometres.

- Multiply by the imperial to metric conversion factor for miles to kilometres.

$123 \times 1.609 = 197.907$
123 mi. = 197.907 km

Converting units of mass

Convert 12 pounds to kilograms.

- Multiply the imperial to metric conversion factor for pounds to kilograms.

$12 \times 0.454 = 5.448$
12 lb. = 5.448 kg

Convert 15 grams to ounces.

- Multiply by the metric to imperial conversion factor for grams to ounces.

$15 \times 0.035 = 0.525$
15 g = 0.525 oz.

Converting units of capacity

Convert 8 U.S. gallons to litres.

- Multiply by the imperial to metric conversion factor for gallons to litres.

$8 \times 3.79 = 30.32$
8 gal. = 30.32 L

Convert 250 mL to pints.

- Convert mL to L.

250 mL = 0.25 L

- Multiply by the metric to imperial conversion factor for pints to litres.

$$0.25 \times 2.11 = 0.53$$
$$0.25 \text{ L} = 250 \text{ mL} = 0.53 \text{ pt.}$$

Now complete the Learning Task Self-Test.

Self-Test 2

1. Perform the following metric conversions.

   a. 2.36 m = _____ mm
   b. 15 000 m = _____ km
   c. 704 mm = _____ m
   d. 1.2 m = _____ mm
   e. 0.53 kg = _____ g
   f. 25 mg = _____ g
   g. 3 kg = _____ t
   h. 0.01 L = _____ mL
   i. 175 mL = _____ cL
   j. 23.1 L = _____ mL

2. Using the chart below, perform the following metric/imperial conversions. Round your answers to the nearest thousandth if necessary.

   a. 24 mm = _____ in.
   b. 18 in. = _____ cm
   c. 30 kg = _____ lb.
   d. 10 oz. = _____ g
   e. 100 lb. = _____ kg
   f. 10 qt. = _____ L
   g. 40 km = _____ mi.
   h. 59.7 L = _____ qt.
   i. 3 gal. = _____ L
   j. 5 t = _____ lb.

	Metric to imperial (U.S.)	Imperial (U.S.) to metric
Length	1 km = 0.621 mi. 1 m = 1.094 yd. 1 m = 3.28 ft. 1 cm = 0.394 in. 1 mm = 0.039 in.	1 mi. = 1.61 km 1 yd. = 0.914 m 1 ft. = 0.305 m 1 in. = 2.54 cm 1 in. = 25.4 mm
Mass	1 kg = 2.20 lb. 1 g = 0.035 oz. 1 t (1000 kg) = 2204.6 lb.	1 oz. = 28.4 g 1 lb.= 0.454 kg
Capacity	1 L = 2.11 pt. 1 L = 1.06 qt.	1 pt. = 0.47 L 1 qt. = 0.95 L 1 U.S. gal. = 3.79 L 1 Imp. gal. = 4.55 L

3. Perform the following metric conversions.

   a. 26.8 m = _____ mm
   b. 6 mg = _____ g
   c. 5 mL = _____ cm^3
   d. 0.68 kg = _____ g
   e. 153 µm = _____ mm
   f. 35.6 km = _____ cm
   g. 0.015 Mg = _____ kg
   h. 7 L = _____ dL
   i. 1.3 cg = _____ kg
   j. 3.29 × 102 m = _____ m

4. Using the chart below, perform the following metric/imperial conversions. Round your answers to the nearest thousandth, if necessary.

a. 12 in. = _____ cm

b. 20 gal. = _____ L.

c. 420 g = _____ oz.

d. 3 ft. = _____ cm

e. 100 kg = _____lb.

f. 100 m = _____ ft.

g. 8 mm = _____in.

h. 82 mi. = _____ km

i. 1000 lb. = _____ t

j. 0.8 L = _____ qt.

	Metric to imperial (U.S.)	Imperial (U.S.) to metric
Length	1 km = 0.621 mi. 1 m = 1.094 yd. 1 m = 3.28 ft. 1 cm = 0.394 in. 1 mm = 0.039 in.	1 mi. = 1.61 km 1 yd. = 0.914 m 1 ft. = 0.305 m 1 in. = 2.54 cm 1 in. = 25.4 mm
Mass	1 kg = 2.20 lb. 1 g = 0.035 oz. 1 t (1000 kg) = 2204.6 lb.	1 oz. = 28.4 g 1 lb.= 0.454 kg
Capacity	1 L = 2.11 pt. 1 L = 1.06 qt.	1 pt. = 0.47 L 1 qt. = 0.95 L 1 U.S. gal. = 3.79 L 1 Imp. gal. = 4.55 L

LEARNING TASK 3

Solve problems involving ratio and proportion

Proportions are very useful in solving problems in everyday life. A proportion is made up of two equal fractions or ratios.

Many of the calculations you will have to do in your trade are likely to be of the kind you can solve by using ratio and proportion. For example, prices and quantities of materials, percentages of figures, forces required to move objects, and number of revolutions per minute can often be quickly calculated or estimated using ratio and proportion. By the same means, plans can be rescaled or gear ratios verified.

Key terms

Cross multiplication: Multiplying the numerator of the first ratio by the denominator of the second ratio, and the denominator of the first ratio by the numerator of the second ratio. If the two products are equal, the proportion is true. For example:

$$\frac{1}{2} = \frac{3}{6} \qquad \begin{array}{l} 1 \times 6 = 6 \\ 2 \times 3 = 6 \end{array}$$

This is a true proportion.

Direct proportion: When one ratio is increased or decreased, it causes another to increase or decrease by the same factor, so the two remain equal.

Inverse proportion: The relation between quantities, the product of which is a constant. When one quantity increases, the other decreases by the same factor, and when one quantity decreases, the other increases by the same factor. For example, *as speed increases, the time required to arrive at a destination decreases proportionately*. Problems in this Competency will involve only direct proportions.

Inverse ratio: A ratio expressed with the terms inverted. For example, the inverse of the ratio 1:2 is 2:1.

Proportion: A statement of equal relationship between two ratios. The ratios can be expressed in one of three ways:

1 is to 2 as 8 is to 16
1:2 = 8:16
1/2 = 8/16

Ratio: A comparison between two quantities, or a way of stating the relationship between them using division. Ratio is expressed in one of four ways:

1 to 2
1:2
1÷2
½

The two numbers are known as the *terms* of the ratio.

Working with ratios

To write a ratio you must identify the quantities to be compared. For example, in a concrete mixture made with 10 parts sand and 2 parts cement, the relationship between the sand and the cement can be expressed as the ratio 10:2.

Like fractions, ratios are normally reduced to their lowest terms. By dividing the terms of the sand and cement ratio by 2, you can express the ratio in lower terms, 5 to 1 (5:1 or 5÷1 or 5/1). In this way, 10:2 and 5:1 are said to be equivalent ratios.

In ratios it is important to be clear about which quantity is being compared to which. In the example above, the ratio of sand to cement is 5:1 (5/1) and the ratio of cement to sand is 1:5 (1/5). It is often helpful to set up the ratio in words before you express it in numbers.

Example
If a shaft turns 14 times per minute, what is the ratio of minutes to turns?

- Write a fractional ratio, using the word that precedes *to* or *per* in the question part of the problem as the fraction's numerator, and the word that follows *to* or *per* as the denominator. Note that minutes comes before *to* in the question part of the problem, even though it comes after *per* earlier in the problem.

  $$\frac{\text{minutes}}{\text{turns}}$$

- Substitute the quantities that correspond to the words:

  $$\frac{1}{14} \text{ or } 1:14$$

Working with proportions

A ratio by itself is not especially useful in solving practical problems. However, two ratios that are equivalent can be written as a *proportion*. That proportion can then be used to solve certain kinds of problems.

Example

Two meshing gears are supposed to turn in the ratio of 1:20. You check the system while it is operating and find that the larger gear is rotating at 325 rpm and the smaller gear is rotating at 6500 rpm. Is the equipment operating correctly?

Solution

1. Express the fractional ratio in words as in the example above.

   $$\frac{\text{large gear}}{\text{small gear}}$$

2. Write the given information as a fractional proportion, making sure that the terms are located to correspond to the relationship you showed in the previous step.

   $$\frac{1}{20} = \frac{325}{6500}$$

3. Cross multiply to see if this is a true proportion.
   (If it is a true proportion both products will be the same, indicating that the equipment is operating correctly.)

   $$\frac{1}{20} = \frac{325}{6500}$$

4. Both products are the same. Therefore, the equipment is operating correctly.

   $$1 \times 6500 = 20 \times 325$$
   $$6500 = 6500$$

Solving word problems with ratio and proportion

You can use ratio and proportion to find an unknown quantity when the other three quantities are known.

Example

The ratio of the teeth on a driver gear to the teeth on a driven gear is 2:5. If the driver gear has 24 teeth, how many teeth does the driven gear have?

Solution

1. Express the fractional ratio in words.

   $$\frac{\text{driver gear}}{\text{driven gear}}$$

2. Write the given information as a fractional proportion, making sure you use the correct relationships. Use the symbol N (or x) to identify the unknown quantity.

   $$\frac{2}{5} = \frac{24}{N}$$

3. Cross multiply.

$$\frac{2}{5} = \frac{24}{N}$$

4. Divide both sides by the number that appears with N.

   2N = 120

5. In this case, divide both sides of the equation by 2.

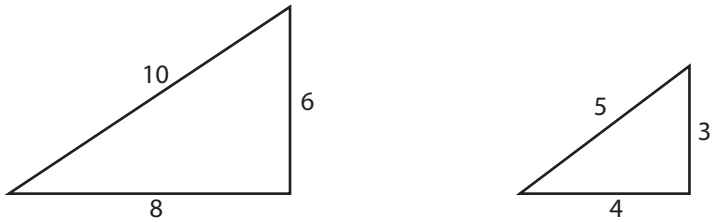
$$\frac{2N}{2} = \frac{120}{2}$$

6. The driven gear has 60 teeth (N = 60).

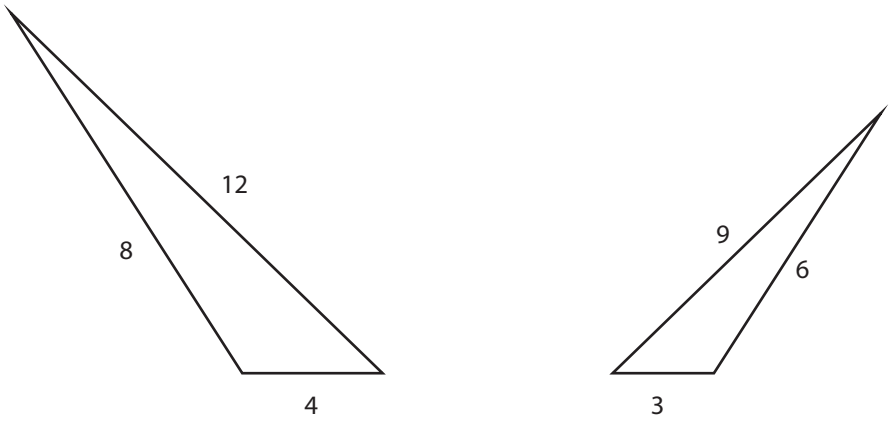
   N = 60

Similar triangles

Two triangles are said to be similar if their corresponding (or matching) angles are equal and the lengths of the corresponding sides are proportional. If we view triangles as having short, middle, and long side lengths, then corresponding sides are the same pair of sides on the respective triangles.



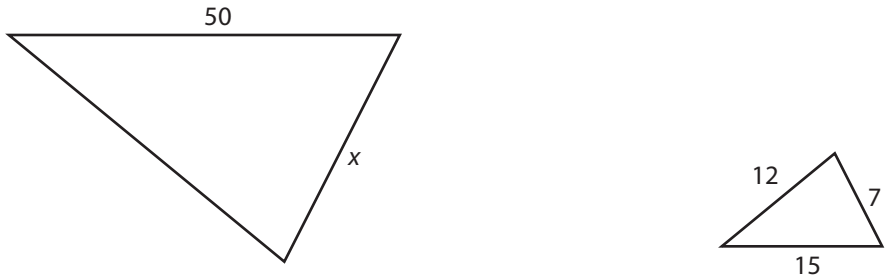
6 corresponds to 3 (short sides)
8 corresponds to 4 (middle sides)
10 corresponds to 5 (long sides)



4 corresponds to 3 (short sides)
8 corresponds to 6 (middle sides)
12 corresponds to 9 (long sides)

Example 1

The triangles are similar. Write a proportion and find *x*.



Solution

Create a ratio from the triangle containing the unknown side, *x*, such as,

$$\frac{x}{50} \quad \frac{\text{short}}{\text{long}}$$

Carefully note that this ratio, $\frac{x}{50}$, compares the short side to the long side in the first triangle.

Set this ratio equal to the $\frac{\text{short side}}{\text{long side}}$ ratio of the other triangle. Then solve the proportion.

$$\frac{\text{short}}{\text{long}} \quad \frac{x}{50} = \frac{7}{15} \quad \frac{\text{short}}{\text{long}}$$

$$15 \times x = 7 \times 50$$

$$15x = 350$$

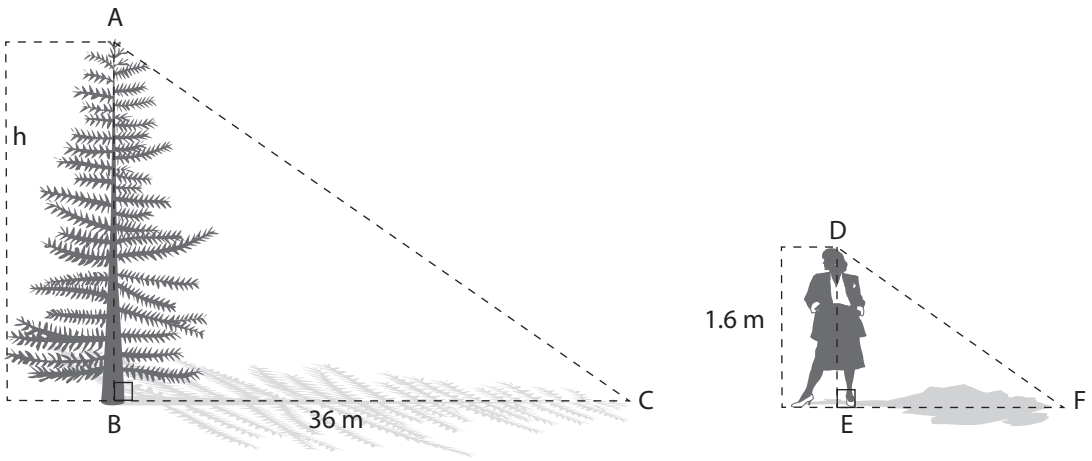
$$x = \frac{350}{15}$$

$$x = 23\tfrac{1}{3} \text{ or } 23.\overline{3}$$

Example 2

Suppose you want to find the height of a tall tree that's too tall to measure directly. You can do this by comparing the length of its shadow cast by the sun to the length of the shadow of a known object.

Since, at any given time, the angle of the sun is the same for all objects in the immediate area, the triangles formed by objects and their shadows are similar.



Solution

In this case, the triangle formed by the tree and its shadow is similar to the triangle formed by the woman and her shadow. The height of the tree is represented by h.

$$\frac{h}{36} = \frac{1.6}{2.4}$$

To find h:

$$2.4 \times h = 36 \times 1.6$$

$$2.4h = 57.6$$

$$h = \frac{57.6}{2.4}$$

$$h = 24$$

Therefore, the tree height must be 24 m.

Scale drawings

Builders, designers, map-makers, etc., use scale drawings to represent real things. Proportions are used to relate the scale drawing to the actual object.

Scale drawings have exactly the same shape as the real object, but they are not always the same size. For example, the drawing of an insect would be larger than the actual insect, and the drawing of a room on a blueprint would be smaller than the actual room that will be built.

When working with scale drawings, you will need to identify the scale used. The scale is the ratio of the drawing measurement to the actual measurement.

When you are using an approximate conversion factor, round your answers to the precision of your approximate factor, if necessary.

Example 1
In a drawing of an insect with a scale of 10:1, a measurement of 10 cm on the drawing represents an actual length of 1 cm.

Find the actual length of the leg of an insect when the leg on the drawing measures 2 cm.

Solution
Set up a proportion to solve the problem. Let n be the actual length of the leg.

$$\frac{\text{drawing measurement}}{\text{actual measurement}} \qquad \frac{10\text{ cm}}{1\text{ cm}} = \frac{2\text{ cm}}{n}$$

$$10 \times n = 2 \times 1$$

$$10n = 2$$

$$n = \frac{2}{10}$$

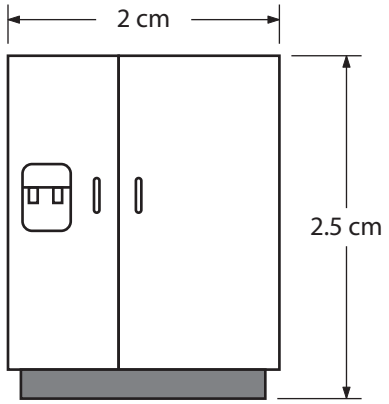
$$n = 0.2$$

The actual length of the leg is 0.2 cm or 2 mm.

Example 2
The scale of 1:40 is used for a drawing of a fridge. Find the actual width and height of the fridge.

Solution
On the drawing, the height of the fridge is 2.5 cm.

Let L be the actual height of the fridge.



$$\frac{\text{drawing measurement}}{\text{actual measurement}} \qquad \frac{2.5}{L} = \frac{1}{40}$$

$$L \times 1 = 2.5 \times 40$$

$$L = 100\text{ cm}$$

The actual height is 100 cm.

On the drawing, the width of the fridge is 2 cm.

so, $\frac{2}{w} = \frac{1}{40}$

$1 \times w = 2 \times 40$

$w = 80$

The actual width is 80 cm.

Now complete the Learning Task Self-Test.

Self-Test 3

1. Express the following ratios in lowest terms:
   a. 15 to 20 = _____
   b. 9 to 3 = _____
   c. 9:63 = _____
   d. 6:1 = _____
   e. 2.4:0.8 = _____

2. Write the inverse of the following ratios:
   a. 5 to 7 = _____
   b. 4/5 = _____

3. Find the missing term in these proportions:
   a. 35:5 = _____ :2
   b. _____ :3 = 28:12

4. Of two gears that mesh, the smaller revolves at 84 rpm and the larger at 36 rpm. In lowest terms, what is the ratio of the speed of the smaller to the larger?
   a. 3:7
   b. 7:3
   c. 5:2
   d. 2:5

5. Two oil tanks have capacities of 1989 L and 1105 L. In lowest terms, what is the ratio of the volume of the larger tank to the smaller?
   a. 3:1
   b. 1:2
   c. 9:5
   d. 6:11

6. The primary winding of an electric transformer has 240 turns, and the secondary winding has 15 turns. The ratio of the primary winding to the secondary is 1:16.
   a. True
   b. False

Use proportion to solve the following problems:

7. A cylindrical barrel contains 9 kL of gasoline when filled to a depth of 3 m. How many kL does the barrel contain when the level is 2 m?
   a. 6 kL
   b. 5 kL
   c. 4 kL
   d. 3 kL

8. In the rear axle of a vehicle, the ratio of teeth on the ring gear to teeth on the pinion gear is 4 to 1. If there are 64 teeth on the ring gear, how many teeth are there on the pinion gear?
   a. 8
   b. 32
   c. 16
   d. 256

9. The ratio of input power to output power of an amplifier is 1:3000. If the input power is 0.00806 W, the amplifier output power will be 24.18 W.
   a. True
   b. False

10. George earns $18.50 an hour; his brother Mike earns $9.25 an hour. What is the ratio in lowest terms of George's hourly wage to Mike's?
    a. 1:2
    b. 2:1
    c. 5:10
    d. 10:5

11. Basement windows in a house are 3000 mm wide by 900 mm high. What is the ratio in lowest terms of width to height?
    a. 3000:900
    b. 900:3000
    c. 3:10
    d. 10:3

12. A starter gear has 12 teeth and a ring gear has 102 teeth. The ratio in lowest terms of the starter gear teeth to the ring gear teeth is 6:52.
    a. True
    b. False

13. The ratio of the crankshaft pulley speed to the alternator pulley speed is 8:19. When the engine crankshaft speed is 736 rpm the speed of the alternator is 1748 rpm.
    a. True
    b. False

14. A steel pipe has a uniform cross-section. The mass of 0.33 m of the pipe is 0.425 kg. What is the mass of a 3.6 m length of the pipe?
    a. 2.795 kg
    b. 4.636 kg
    c. 3.867 kg
    d. 5.385 kg

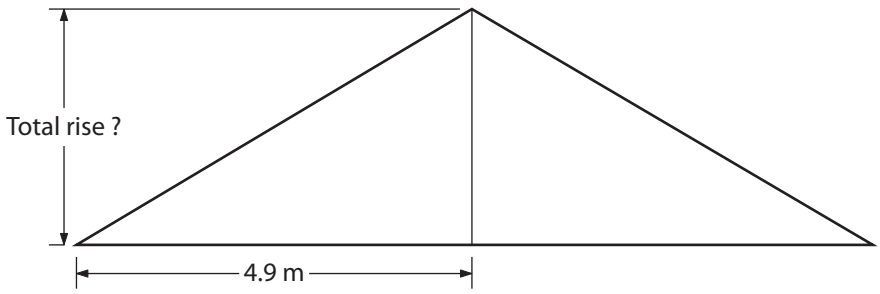
15. You are told to construct a workbench for which the length is 4 1/2 times the width. If your bench is to be 1800 mm long, it must be made 450 mm wide.
    a. True
    b. False

16. In a transformer, the turns ratio varies directly with the voltage ratio. In a power transformer in which the ratio of primary turns to secondary turns is 10:3, what is the secondary voltage when the primary voltage is 123 V?
    a. 41 V
    b. 36.9 V
    c. 410 V
    d. 1230 V

17. In the following chart, A varies directly in proportion to B. Complete the chart.

A	7	14	
B		42	30

18. The slope of a roof is the ratio of unit rise to the unit of run. If the slope is 4:12 and the total run is 4.9 m, the total rise must be 1.633 m.
    a. True
    b. False



19. In mixing concrete for a foundation slab, the ratio of gravel to cement is 7:2. If you estimate you need 12 m^3 of gravel, how much cement powder will you need?
    a. 42 m^3
    b. 3.8 m^3
    c. 3.4 m^3
    d. 4.2 m^3

20. A water pump discharges 50 L of water in 3.5 minutes. How long does it take to discharge 11 000 litres?
    a. 770 min.
    b. 63 min.
    c. 77 min.
    d. 550 min.

21. If 600 kg of carbon steel alloy contains 4.80 kg of manganese, there would be 6.92 kg of manganese in 865 kg of the same steel alloy.
    a. True
    b. False

22. The capacity of an oil reservoir is directly proportional to the depth of oil. If the reservoir holds 280 000 L of oil when filled to its maximum depth of 3.8 m, how much oil is there when the depth is 1.2 m?
    a. 23 333 L
    b. 37 255 L
    c. 66 398 L
    d. 88 421 L

23. A 406 mm shaft has a uniform taper of 0.05 mm in 101.5 mm. What is the taper in the entire length of the shaft?
    a. 0.2 mm
    b. 12 mm
    c. 33 864 mm
    d. 824 810 mm

24. The power of a gas engine increases with the area of the piston head. If an engine with a piston area of 8.3 in.^{2} develops 25.75 horsepower, to the nearest horsepower, how much horsepower will an engine have with a piston area of 25 in.^{2}?
    a. 8 hp
    b. 77 hp
    c. 78 hp
    d. 92 hp

25. If 3 L of paint covers a surface containing 82.5 m^2, how much paint will be needed to cover 192.5 m^2 of surface?
    a. 7 L
    b. 12 L
    c. 27.5 L
    d. 5293.75 L

LEARNING TASK 4

Solve problems involving percent

Percent, or the symbol %, is very common in everyday life. For example, sales may be advertised as 25% or 50% off; sales tax is 7%; loans can be made with interest charges of anywhere from 1% to 20%. The word *percent* means *per hundred*. The word *cent* comes from the Latin word *centum,* which means *hundred*.

Applications for percentage in the trades include measuring efficiency of equipment, calculating proportions of mixtures, and calculating tolerances. Knowing how to calculate discounts and increases is important for equipment and material purchase, and essential if you plan to run your own business.

Percents (or percentages as they are more correctly called) are closely related to fractions and decimals.

7% means 7 per hundred

$\frac{7}{100}$ means 7 per hundred

0.07 means 7 per hundred

so

$$7\% = \frac{7}{100} = 0.07$$

To change from percent notation to fraction or decimal notation, simply replace the percent sign with $\frac{1}{100}$ or 0.01.

Converting between decimals and percents

The symbol % does the work of two decimal places. Thus, 59% is equivalent to 0.59 and 0.172 is equivalent to 17.2%.

The "D - P Rule" is a simple way to remember which way to move the decimal point:
D comes before *P* in the alphabet, so it is always written first.

Note the use of zeros as placeholders in the above examples.

To change percents to decimals, go left two places.

D ← P 5% = 0.05

To change decimals to percents, go right two places.

D → P 0.4 = 40%

Examples of converting between decimals and percents

Example 1
Convert 7.25%, 136%, and 16¾% to decimals.

Solutions

1. 7.25% = 7.25 × 0.01 = 0.0725 (move the decimal two places to the left)

   7.25% = 0.0725

2. 136% = 136 × 0.01 = 1.36 (move the decimal two places to the left)

   136% = 1.36

3. 16¾% = 16.75 × 0.01 = 0.1675 (convert the fraction ¾ to the decimal 0.75, then move the decimal two places to the left)

   16.75% = 0.1675

Example 2
Convert 0.386, 2.45, and 0.008 to percent.

Solutions

1. 0.386 = 0.386 × 100% = 38.6% (move the decimal two places to the right)

   0.386 = 38.6%

2. 2.45 × 100% = 245% (move the decimal two places to the right)

   2.45 = 245%

3. 0.008 = 0.008 × 100% = 0.8% (move the decimal two places to the right)

   0.008 = 00.8%

Converting between fractions and percents

Percent means *per hundred*. The percent symbol (%) following a number means that quantity out of a hundred. Since percent means per hundred, 100% of something means 100/100 or 1. Thus, 100% of 55 is 55. Seven percent (7%) means seven-hundredths, seven out of a hundred, or 7/100.

Percent to fractions

To convert a percentage to a fraction, you put the percentage over a denominator of 100 and reduce to lowest terms, if necessary. For example:

$$36\% = \frac{36}{100} = \frac{9}{25}$$

Not all conversions are this straightforward. For example:

$$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100}$$
$$= 33\frac{1}{3} \div 100$$
$$= \frac{100}{3} \div \frac{100}{1}$$
$$= \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$$

Example 1

Convert 12%, $\frac{2}{5}\%$, $16\frac{2}{3}\%$, and 12.5% to fractions.

Solutions

1. $12\% = 12 \times \frac{1}{100} = \frac{12}{100} = \frac{3}{25}$
2. $\frac{2}{5}\% = \frac{2}{5} \times \frac{1}{100} = \frac{2}{500} = \frac{1}{250}$
3. $16\frac{2}{3}\% = \frac{50}{3}\% = \frac{50}{3} \times \frac{1}{100} = \frac{50}{300} = \frac{1}{6}$
4. $12.5\% = \frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}$

Fractions to percent

Converting a fraction to a percent involves multiplying the numerator by 100 and dividing this product by the denominator.

- Convert 5⁄16 to a percent.
- Multiply the numerator by 100.

  $$\frac{5}{16} \times 100 = \frac{500}{16}$$

- Divide this product by the denominator.

  500 ÷ 16 = 31.25 (or 31¼)

- Add a percent symbol to your quotient.

  31.25% (or 31¼%)

Example 1

Convert $\frac{3}{25}$, $\frac{1}{250}$, $\frac{1}{6}$ and $\frac{1}{8}$ to percents.

Solutions

1. $\frac{3}{25} \times 100 = \frac{300}{25}$
   $300 \div 25 = 12$
   12%

2. $\frac{1}{250} \times 100 = \frac{100}{250}$
   $100 \div 250 = 0.4$
   0.4%

3. $\frac{1}{6} \times 100 = \frac{100}{6}$
   $100 \div 6 = 16.\overline{6}$ (or 16⅔)
   16.67% (or 16⅔%)

4. $\frac{1}{8} \times 100 = \frac{100}{8}$
   $100 \div 8 = 12.5$
   12.5%

Solve percent equations and word problems

You can use proportions to solve three types of percent problems.

The proportion can be used to find the:

1. percent when the part and total are known
2. part when the percent and total are known
3. total when the part and percent are known

Total can also mean the whole amount or the final amount.

$$\frac{\text{part}}{\text{total}} = \frac{\text{percent}}{100\%}$$

Any percent problem can be written using the following pattern:

_____ is _____ % of _____ .

The following examples demonstrate how to apply the two known pieces to solve any percent problem.

Example 1

If a mechanic has completed 6 out of the 30 hours required for a job, what percent of the job has been completed?

Solution

- Rephrase the question to fit the given pattern statement if necessary.

  6 is what % of 30

- Substitute the following symbols in your pattern statement: "=" for "is"; "N" for "what"; "×" for "of."

  $6 = N\% \times 30$

- Change the percent to a fraction with 100 as its denominator.

  $$6 = \frac{N}{100} \times 30$$

- Divide both sides of the equation by 30 to set up as a proportion. (See Learning Task 1 if you need to review this skill.)

  $$\frac{6}{30} = \cancel{30} \times \frac{N}{100} \times \frac{1}{\cancel{30}}$$

  $$\frac{6}{30} = \frac{N}{100}$$

- Solve as a proportion by multiplying each side by 100 to solve for N.

  $$\frac{10\cancel{0}}{1} \times \frac{6}{3\cancel{0}} = \frac{N}{\cancel{100}} \times \frac{\cancel{100}}{1}$$

  N = 20

- Insert your answer in the pattern statement.

  6 is 20% of 30

Example 2
A certain alloy is 60% tin. How much tin is contained in 42 kilograms of this alloy?

Solution

- Rephrase to fit the given pattern statement.

  what is 60% of 42

- Substitute the symbols as in the example above.

  $N = 60\% \times 42$

- Change the percentage to a fraction.

  $$N = \frac{60}{100} \times 42$$

- Divide both sides of the equation by 42 to set up as a proportion and solve.

  $$\frac{N}{42} = \frac{60}{100};\ N = 25.2$$

- Insert your answer into the pattern statement.

  25.2 is 60% of 42

Example 3
An article loses 17 mm in machining. This is 5% of its original size. What was the original size?

Solution

- Rephrase and substitute symbols.

  $17 = 5\% \times N$

- Change the percentage to a fraction.

  $$17 = \frac{5}{100} \times N$$

- Divide both sides of the equation by N to set up as a proportion and solve.

  $$\frac{17}{N} = \frac{5}{100};\ N = 340$$

- Insert your answer into the pattern statement.

A simple way to remember the above procedure is to memorize this equation:

$$\frac{\%}{100} = \frac{IS}{OF}$$

Once you have rephrased your questions to fit the pattern statement, put the number that follows *of* in the equation's OF location, the number that precedes the percent symbol in the % location, and the remaining number in the IS location.

Using a calculator's percent function to solve problems

The "%" button on your calculator can be used to find what percent one number is of another number. The following examples demonstrate the use of this function on your calculator.

Example 1

Out of 1757 motors, 7% were rejected by inspectors. How many motors were rejected?

Solution

- Rephrase to fit the pattern statement and substitute symbols.

  $N = 7\% \times 1757$

- Enter the number that is to be multiplied by the known percentage.

  1757

- Press "×" on the calculator, enter the known percentage and press "%."

  × 7%

- Your answer will appear immediately.

  123

Example 2

A certain ore yields 3.5% of iron. How many tons of ore are required to produce 150 tons of iron?

Solution

- Rephrase to fit the pattern statement and substitute symbols.

  $150 = 3.5\% \times N$

- Divide both sides of the equation by 3.5%, and cancel to isolate the N.

  $$\frac{150}{3.5\%} = \frac{\cancel{3.5} \times N}{\cancel{3.5\%}}$$

  $$\frac{150}{3.5\%} = N$$

- Enter 150, press "÷," enter 3.5, and press "%."

  150 ÷ 3.5%

- Your answer will appear immediately.

  4285.7142

Adding and subtracting percents

There are many cases where percents must be added or subtracted.

Example 1

The deductions to Frank's paycheque amounted to 36.2%. What percent of the paycheque did Frank take home?

Solution

The total paycheque must be represented as 100% of the paycheque.

100% – 36.2% = 63.8%

Example 2

Sulphuric acid by weight contains 2% hydrogen and 65% oxygen. The rest is sulphur. What percent of sulphuric acid is sulphur?

Solution

The total weight must be represented by 100%.

100% – (2% + 65%) = 100% – 67% = 33%

Common percent applications

Sales tax

Percent is widely used in sales tax computations. The provincial sales tax (PST) rate in British Columbia in 2015 is 7%. This means that the tax is 7% of the purchase price. Another sales tax, the goods and services tax (GST) is collected by the federal government and is 5% of the purchase price. The total price of an article is the price plus PST and GST.

Example

A refrigerator is priced at $699.95 plus PST and GST. How much is the PST, the GST, and the total price?

Solution

Find the PST:

$$\frac{x}{699.95} = \frac{7}{100}$$
$$100x = 699.95 \times 7$$
$$100x = 4899.65$$
$$x = 48.996$$

Round to the nearest cent = $49.00

Now find the GST:

$$\frac{x}{699.95} = \frac{5}{100}$$
$$100x = 699.95 \times 5$$
$$100x = 3499.75$$
$$x = 34.997$$

Round to the nearest cent = $35.00

The total price is $699.95 + $49.00 + $35.00 = $783.95

Another way to calculate the percentage is to look at it as multiplying your value by the decimal equivalent of the percentage, which can easily be calculated by moving the decimal two places to the left.

$$x = 699.95 \times \frac{7}{100}$$
$$x = 699.95 \times .07$$
$$x = 48.996$$

Simple interest

Whenever you open a savings account or buy a Canada Savings Bond, the bank or the government agrees to pay you interest for the use of your money. If you take out a loan with a bank, they expect you to pay them interest. The rate of the interest paid is always given as a percent.

For example, if you buy a $100.00 Canada Savings Bond that pays 9% interest, then exactly one year later, the government will pay you $9.00 interest on your bond, because 9% of $100.00 = $9.00. Of course if you decide to cash in the bond before the year is out, then the interest will be less than $9.00.

To find simple interest, use the following formula:

$$I = Prt$$

where:

I = interest earned in dollars
P = principal amount that earns the interest in dollars
r = interest rate per year in percent
t = time in years

Example

Mary has a $500 GIC (guaranteed investment certificate) that earns interest at a rate of 8½% over three years. How much simple interest will it earn over three years?

Solution

Rewrite the percent number as a decimal:

8½% = 8.5% = 0.085

Write the simple interest formula using the information:

$I = P \times r \times t$
$I = 500 \times 0.085 \times 3$

Multiply and remember to write the answer in dollars and cents:

$I = 127.5$

The interest is $127.50.

Discounts and increases

The simplest way to figure out discounts and increases is to rephrase the problem before you begin. For example, if you are asked to calculate a 15% increase in cost, you would call the present cost 100%, add the 15% and call the new cost 115%. The following examples demonstrate how to calculate discounts and increases.

Example 1

If an article was priced at $189.50 and is now marked down 20%, what is its new price?

- Subtract the discount from 100%, rephrase to fit the pattern statement, and substitute symbols.

  N = 80% × $189.50

- Solve.

  $151.60

Example 2

Over 15 years, the cost of an item has increased 73%. The original cost was $58.47. What is the new cost?

- Add the increase to 100%, rephrase to fit the pattern statement and substitute symbols.

  N = 173% × $58.47

- Solve.

  $101.15

Here are some mental strategies for finding percentages:

- To find 50%, find half .
- To find 25%, find half and half again.
- To find 75%, find half and half again, and add them together .
- To find 10%, divide by 10
- To find 5%, find 10% and halve it.
- To find 20%, find 10% and double it.

Now complete the Learning Task Self-Test.

Self-Test 4

1. Fill in the chart below with the required numbers. Provide all fractions in lowest terms.

	Fraction	Decimal	Percent
a.		0.62	
b.			8%
c.	2/9		
d.		0.075	
e.	1 4/5		
f.			95%

2. _____ is 6% of 63.

3. 9 is _____% of 27.

4. 84 is 375% of _____.

5. 48 is 20% less than _____.

6. 40% of 24 is what?
   a. 14.4
   b. 9.6
   c. 8.4
   d. 10.2

7. What percent of 1.32 is 0.6?
   a. 46%
   b. 55%
   c. 45%
   d. 10%

8. 3 is 15% of 22.
   a. True
   b. False

9. 30 is 20% more than what?
   a. 25
   b. 24
   c. 22
   d. 20

10. 102 is 15% less than what?
    a. 115
    b. 120
    c. 124
    d. 122.4

11. 63 is what percent of 40, to the nearest percent?
    a. 63%
    b. 157%
    c. 158%
    d. 145%

12. 12% of 30 is 3.6.
    a. True
    b. False

13. 4.06 is 9% of 48.
    a. True
    b. False

14. One part acid and four parts water are mixed as an electrolyte for a storage battery. What percent of the electrolyte is acid?
    a. 25%
    b. 20%
    c. 75%
    d. 80%

15. The efficiency of a motor is 90%. If the motor delivers 13.5 horsepower, what is the input?
    a. 15 hp
    b. 14.85 hp
    c. 12.15 hp
    d. 15.28 hp

16. A set of combination wrenches priced at $179.99 went on sale for 15% off. What is the sale price?
    a. $26.99
    b. $99.99
    c. $152.99
    d. $159.99

17. A company has recently increased its workforce by 12½%. There are now 18 employees in the company. How many workers were there before the increase?
    a. 12
    b. 14
    c. 16
    d. 18

18. The operating spindle speed of a lathe spindle that rotates freely is 354 rpm. When 16% is lost through slippage and cutting pressure, the new speed is 297 rpm.
    a. True
    b. False

19. A floor joist is allowed to vary 5% of its stated width of 184 mm. The maximum width would be what?
    a. 200 mm
    b. 188.4 mm
    c. 193.2 mm
    d. 197.9 mm

20. What percent is wasted when 3.6 of every 120 sheets of metal are spoiled?
    a. 3%
    b. 0.3%
    c. 30%
    d. 300%

21. The minimum idle speed for a particular engine is 630 rpm. This is 94% of the maximum idle speed of 668 rpm.
    a. True
    b. False

22. A contractor wants to make a profit of 12% on a job. If the break-even cost is $2345, what should the total job price be, including the profit?
    a. $2626.00
    b. $2600.40
    c. $2600.00
    d. $2626.40

LEARNING TASK 5

Solve problems involving powers, roots, and scientific notation

Key terms

Base: In an expression of a number to a power, the base is the number that is to be multiplied by itself. For example, in 8^3, the base is 8.

Cube: The result of a number being multiplied by itself three times. For example, the cube of 4 (4 cubed) is $4^3 = 4 \times 4 \times 4 = 64$.

Cube root: The cube root of a number is the value that, when multiplied by itself three times, results in the number. The cube root of 125 is 5. Read $\sqrt[3]{125}$ as the "cube root of 125."

Exponent: A small number written above and to the right of a base. The exponent indicates how many times the base is to be multiplied by itself; also known as the *degree* of a power.

Index: The number written to the left and above the root sign; it indicates what root is to be taken. If the root sign has no index, it is understood that the square root is to be taken. The index 3 in $\sqrt[3]{8}$ indicates that the cube root of 8 is to be taken.

Power: The product that results from multiplying a base by itself the number of times indicated by the exponent. In the power 7^3, 7 is the base; 3 is the exponent. This power is read "seven to the third" or "seven cubed." It means $7 \times 7 \times 7$.

Radical: An expression involving the root symbol $\sqrt{\ }$.

Radicand: The number inside the root (or radical symbol). In the expression $\sqrt[3]{5}$, 5 is called the radicand.

Root: The root of a number is the value that, when multiplied by itself the indicated number of times, results in the number.

Root sign: The symbol $\sqrt{\ }$ is the root sign.

Scientific notation: A shorter method of writing very large or very small numbers, using a power of 10 as a multiplier.

Square: The result of a number being multiplied by itself. For example, the square of 3 (3 squared, or 3×3) is 3^2 or 9.

Square root: The square root of a number is the value that, when multiplied by itself, results in the number. The square root of 9 is 3.

Important concepts

1. Any number to the first power (having an exponent of 1) is equal to itself.
   $6^1 = 6$

2. Any number to the zero power (having an exponent of 0) equals one.
   $8^0 = 1$

3. Any number to a power between 0 and 1 will be less than the base.
   $16^{0.5} = 4$

4. Any number to a power greater than 1 will be more than the base.
   $7^2 = 49$

Powers

As you may recall, multiplying is a way of indicating how many times a number should be added. For example:

$5 \times 3 = 5 + 5 + 5$

The power operation is a way of indicating how many times a number should be multiplied by the same number. For example:

$5^3 = 5 \times 5 \times 5$

The power operation is indicated by placing a small number, called an *exponent*, to the top right of some number, or base.

5^3 is called a *power*.
5 is called the *base*.
^{3} is called the *exponent*.

The power 5^3 is read "five to the third" or, more commonly, "five cubed." Powers with exponents of 2 are said to be "squared," while those with exponents of 3 are said to be "cubed." Below are some examples of how to read powers.

7^2 is read "seven squared."
7^3 is read "seven cubed."
9^4 is read "nine to the fourth."
12^8 is read "twelve to the eighth."

Notice that the exponent is not really a number—it is an instruction that says, "multiply the number below me by itself this many times." So $9^4 = 6561$, not 36.

You can use exponents to find powers of integers, decimals, and fractions:

$(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

$(1.25)^3 = (1.25)(1.25)(1.25) = 1.953125$

$$\left(\frac{2}{3}\right)^8 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{256}{6561}$$

You probably realize now why exponents are used; they save a lot of time writing down repeated multiplication.

Scientific calculators have a "raise to the power" key, usually labelled y^x (sometimes x^y or a^x is used). To use this function, press the y^x key in between the base number and the exponent, then press = .

Example 1

To find $3^5 = ?$ press: 3 [y^x] 5 [=]

Did you get 243?

Example 2

To find $(1.25)^3 = ?$ press: 1.25 [y^x] 3 [=]

Did you get 1.953125?

Example 3

To find $(2.57)^{10} = ?$ press: 2.57 [y^x] 10 [=]

Did you get 12569.88294?

Depending on what kind of calculator you have, you may find that you cannot use a negative number as a base. For example, $(-3)^5$ may result in an error message on the screen. If this is the case, you can raise the positive number to that power, and then make your answer positive or negative. Note that pairs of negative signs will cancel out to give positive answers:

$(-3)(-3) = 9$ $(-3)(-3)(-3) = -27$ $(-3)(-3)(-3)(-3) = 81$ so $(-3)^5 = -243$

Something else to remember: when the base is negative, the negative sign is not part of the number that is raised to the power.

$(-5)^4 = (-5)(-5)(-5)(-5) = 625$
$-5^4 = -1 \times 5^4 = -1 \times 5 \times 5 \times 5 \times 5 = -625$

In the first example, the negative is part of the 5, but not part of the 4th power. Think of it as a negative one multiplied by 5^4, and remember that multiplying takes place after powers have been calculated.

Take a look at the following patterns:

$2^4 = 2 \times 2 \times 2 \times 2$	$= 16$	$10^4 = 10 \times 10 \times 10 \times 10$	$= 10000$
$2^3 = 2 \times 2 \times 2$	$= 8$	$10^3 = 10 \times 10 \times 10$	$= 1000$
$2^2 = 2 \times 2$	$= 4$	$10^2 = 10 \times 10$	$= 100$
2^1	$= 2$	10^1	$= 10$
2^0	$= 1$	10^0	$= 1$

Notice that each line is equal to one factor (2 or 10) divided into the line above it. It seems reasonable the $2^1 = 2$ or $10^1 = 10$. After all, the exponent gives the number of factors that are multiplied together. But why should $2^0 = 1$ or $10^0 = 1$? A zero exponent does look odd, but it has to follow the same pattern:

$2^3 \div 2 = 2^2 = 4$	$10^3 \div 10 = 10^2 = 100$
$2^1 \div 2 = 2^0 = 1$	$10^1 \div 10 = 10^0 = 1$

Note that your calculator can use zero as an exponent, except in the case 0^0, which is undefined. Otherwise, any base number raised to the 0 power equals 1.

Finally, here is a shortcut you can use on your calculator: The most common exponent used in algebra is 2. Your calculator will have a key labelled x^2. You can use this key instead of the y^x key, and save yourself some time:

13 $\boxed{x^2}$ or 13 $\boxed{x^2}$ $\boxed{=}$

Did you get 169?

Fractions can also be squared as follows:

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$$

To square the 2 and 3 separately:

2 $\boxed{x^2}$ is 4 and 3 $\boxed{x^2}$ is 9

The fraction squared can be rewritten as:

$$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Application of powers

The following formulas all contain powers. They are examples from geometry, science, and finance.

Example 1: Area of a circle

$A = \pi r^2$

where:

A = area

r = radius

$\pi = 3.1415927$

Find the area of a circle when r = 3 cm. (Round the answer to one decimal place.)

Solution

$A = \pi (3)^2$

$A = \pi (9)$

$A = 28.3 \text{ cm}^2$

Example 2: Volume of a sphere

$$V = \left(\frac{4}{3}\right)\pi r^3$$

where:

V = volume

$\pi = 3.1415927$

r = radius

Find the volume of a sphere when r = 5 cm. (Round the answer to one decimal place.)

Solution

$$V = \left(\frac{4}{3}\right)\pi(5)^3$$

$$V = \left(\frac{4}{3}\right)\pi(125)$$

$V = 523.6 \text{ cm}^3$

Example 3: Power required to overcome air resistance

$$\text{Power required in kilowatts} = \frac{S^3A}{23\,400}$$

where:

S = speed in km/hr

A = frontal area in m^2

23 400 is a constant for air resistance calculations

Find the power required to overcome air resistance when S = 40 km/hr and A = 5 m^2. (Round the answer to one decimal place.)

Solution

$$P = \frac{(40)^3(5)}{23\,400}$$

$$P = \frac{(64\,000)(5)}{23\,400}$$

$$P = 13.7 \text{ kilowatts}$$

Example 4: Gravitational force of Earth

$$F = \frac{(400\,000\,000)\ M}{d^2}$$

where:

F = gravitational force in newtons
M = mass in kg
d = distance from Earth's centre in km
400 000 000 is a constant for Earth's gravity calculations

Find Earth's gravitational force on a 70 kg person when they are 6500 km from the centre of Earth. (Round the answer to one decimal place.)

Solution

$$F = \frac{(400\,000\,000)(70)}{(6500)^2}$$

$$F = \frac{(400\,000\,000)(70)}{42\,250\,000}$$

$$F = 662.7 \text{ newtons}$$

Example 5: Compound interest

Compound interest is the interest calculated not only on the original principal, but also on all interests calculated in previous years.

$$A = P(1 + r)^t$$

where:

A = total amount
P = principal amount
r = annual interest rate as a decimal
t = years

You have bought a Canada Savings Bond for $800 at an interest rate of 3.5%. In two years' time you cash it in. What is the total amount you will receive?

Solution

$A = (800)(1 + .035)^2$
$A = 800(1.035)^2$
$A = 800(1.071225)$
$A = \$856.98$

Square roots

A root is the reverse of a power. Since $5^2 = 25$, we say that 25 is the square of 5. We could reverse this statement, and say that 5 is the square root of 25. In symbols, this would be written:

$$\sqrt{25} = 5$$

The symbol $\sqrt{\ }$ means "find the number, that when squared, equals the number inside." Expressions involving the symbol $\sqrt{\ }$ are called *radical expressions* and the number inside it is called the *radicand*. The radicand in the expression $\sqrt{25}$ is 25. The expression $\sqrt{25}$ reads "the square root of 25."

Take a look at the following:

$\sqrt{9} = 3$ because $3 \times 3 = 9$

$\sqrt{49} = 7$ because $7 \times 7 = 49$

$\sqrt{100} = 10$ because $10 \times 10 = 100$

$\sqrt{0.25} = 0.5$ because $0.5 \times 0.5 = 0.25$ (one-half of a half)

$\sqrt{\frac{1}{16}} = \frac{1}{4}$ because $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ (one-quarter of a quarter)

What about numbers that do not have whole number square roots? For example, what is the square root of 30?

Look for the $\boxed{\sqrt{\ }}$ button on your calculator. Use this button to find the above square roots. Did you get the right answers? Try the following, using your $\boxed{\sqrt{\ }}$ button.

$\sqrt{2}$ = _____ Did you get 1.414213562?

$\sqrt{195}$ = _____ Did you get 13.96424004?

$\sqrt{0.05}$ = _____ Did you get 0.223606797?

$\sqrt{\frac{2}{3}}$ = _____ Did you get 0.81649658?

$\sqrt{-4}$ = _____ Did you get ERROR?

Apparently, the $\sqrt{-4}$ has no answer. This is because there is no number that can be multiplied by itself to give –4. Note that $2 \times 2 = 4$ and $-2 \times -2 = 4$, and even though $2 \times -2 = -4$, 2 and –2 are not the same number.

Try entering $-\sqrt{4}$ Did you get –2? This is the method used to identify that the root is negative.

- **$\sqrt{A}$ = b because b × b = A.**
- **A must be a positive number.**
- **b can be a positive or negative number. It is assumed that b is positive unless a negative sign appears in front of the root symbol.**
- **The square root of a negative number is undefined.**

So the $\sqrt{-25}$ is undefined. Since $-5 \times -5 = 25$, we would have to ask for the negative root of 25: $-\sqrt{25} = -5$.

Indexed radicals

Radicals can be used to represent other kinds of roots as well, by using an index number to indicate a cube (3rd) root, 4th root, or any higher root. For example:

$$\sqrt[5]{32} = 2$$

The above statement would be read as: "the 5th root of 32 equals 2." Now examine the following:

$\sqrt[5]{32} = 2$, since $2 \times 2 \times 2 \times 2 \times 2 = 32$ or $2^5 = 32$

$\sqrt[3]{27} = 3$, since $3^3 = 27$

$\sqrt[8]{65536} = 4$, since $4^8 = 65\,536$

$\sqrt{81} = 9$, since $9^2 = 81$

No index number is used when it is a square root (index number = 2).

To find a root with a calculator, you need to use the $\sqrt[x]{\ }$ or $\sqrt[x]{y}$ key. This function is usually above one of the other keys (often the y^x key). This means that you will probably have to press the 2nd function key (sometimes labelled SHIFT or INV) first. Note also that some calculators that use direct algebraic logic (DAL) require a different sequence of key entries than older models (or Texas Instrument (TI) style calculators).

Key entries for TI-style calculator

To find $\sqrt[5]{32}$ = ?	press 32	$\boxed{2^{nd}\ f}$	$\boxed{y^x}$	5	$\boxed{=}$	2
To find $\sqrt[3]{216}$ = ?	press 216	$\boxed{2^{nd}\ f}$	$\boxed{y^x}$	3	$\boxed{=}$	6

Key entries for DAL-style calculator

To find $\sqrt[5]{32} = ?$	press 5	$\boxed{2^{nd}\ f}$	$\boxed{y^x}$	32	$\boxed{=}$	2
To find $\sqrt[3]{216} = ?$	press 3	$\boxed{2^{nd}\ f}$	$\boxed{y^x}$	216	$\boxed{=}$	6

Write down here the key entries that you would use on your own calculator: _____ .

Note that the answer to $\sqrt[4]{10}$ is approximately 1.77828. Most roots will be irrational numbers, rather than whole numbers or fractions.

A root can also be written as a power, using a fractional exponent.

$$\sqrt[5]{32} = 32^{\frac{1}{5}} = 2$$

$$\sqrt[3]{8} = 8^{\frac{1}{3}} = 2$$

This means that a root can also be found using the y^x key:

To find $32^{\frac{1}{5}} = ?$	press 32	$\boxed{y^x}$	$\boxed{(}$	1	$\boxed{\div}$	5	$\boxed{)}$	$\boxed{=}$	2
To find $81^{\frac{1}{4}} = ?$	press 81	$\boxed{y^x}$	$\boxed{(}$	1	$\boxed{\div}$	4	$\boxed{)}$	$\boxed{=}$	3

Scientific notation

Remember this sequence?

$10^4 = 10 \times 10 \times 10 \times 10 \quad = 10000$
$10^3 = 10 \times 10 \times 10 \quad = 1000$
$10^2 = 10 \times 10 \quad = 100$
$10^1 \quad = 10$
$10^0 \quad = 1$

We can continue this sequence using negative exponents:

$$10^{-1} = \frac{1}{10} \quad = 0.1$$

$$10^{-2} = \frac{1}{100} \quad = 0.01$$

$$10^{-3} = \frac{1}{1000} \quad = 0.001, \text{ and so on.}$$

Since we use a base 10 counting system and a base 10 measurement system (SI, or the metric system), this sequence of *powers* of 10 is very useful.

Very large numbers can be expressed by multiplying a decimal number by a power of 10. For example:

15 trillion = 15 000 000 000 000 = 1.5×10^{13}
260 billion = 260 000 000 000 = 2.6×10^{11}

Note that this method of writing a number, or scientific notation, uses a decimal number between 1 and 10 followed by the "x 10" and the appropriate exponent of 10.

The exponent tells you the number of places and the direction that the decimal point would have to be moved.

Example 1

$5.13 \times 10^5 = 5.13 \times 10 \times 10 \times 10 \times 10 \times 10 = 513\ 000$

Notice that the decimal point moves five places to the right.

A negative exponent above the 10 does exactly the same thing for very small numbers. In this case, the negative exponent tells you how many places to move the decimal to the left.

Example 2

$$7.031 \times 10^{-6} = 7.031 \times \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = 0.000\ 007\ 031$$

Scientific calculators usually have a "sci" mode, which will make all of your answers automatically appear in scientific notation. This function may also be used to convert decimals and whole numbers into scientific notation equivalents. Read your own calculator manual to see how this is done.

When entering a number in scientific notation into a calculator, you use the (enter exponent) key [EE] or [EXP].

For example, if you wanted to enter the mass of Earth (5.98×10^{21} tonnes), enter as follows:

5.98 [EE] 21 or 5.98 [EXP] 21, depending on your calculator.

The [EE] or [EXP] replaces the need to press "× 10".

Most calculators will show the exponent in smaller digits. (Some calculators even include the "10.") If your model does not, it will show a space between the decimal part of the number and the exponent.

By now, you should be comfortable dealing with exponents, roots. and scientific notation and be able to carry out these kinds of operations with your calculator.

Now complete the Learning Task Self-Test.

Self-Test 5

1. In the power 5^2, 5 is the base and 2 is the exponent.
   a. True
   b. False

2. $(2/5)^2 =$
   a. 4/5
   b. 4/25
   c. 2/25
   d. 2/5

3. $4^3 =$
   a. 12
   b. 16
   c. 64
   d. 24

4. $22^0 = 0$
   a. True
   b. False

5. $5^1 = 5$
   a. True
   b. False

6. $4^2 + 2^3 =$
   a. 16
   b. 24
   c. 32
   d. 28

7. $(3/8)^2 =$
   a. 0.140
   b. 0.141
   c. 0.142
   d. 0.014

8. $8^3 = 512$
   a. True
   b. False

9. $12^3 =$
   a. 36
   b. 144
   c. 288
   d. 1728

10. $\sqrt{3} = 1.37$
   a. True
   b. False

11. $\sqrt[3]{8} =$
   a. 2
   b. 2.33
   c. 2.25
   d. 2.67

12. $\sqrt{324} =$
   a. 12
   b. 24
   c. 18
   d. 18.49

13. $\sqrt[3]{343} = 6$
   a. True
   b. False

14. $\sqrt{49/81} =$
   a. 0.086
   b. 0.778
   c. 0.777
   d. 0.078

15. $\sqrt{1.21} = 1.1$
   a. True
   b. False

LEARNING TASK 6

Solve problems using equations and formulas

Using simple equations and formulas to solve problems is a useful skill for any trade. You can use this skill to convert temperatures between Fahrenheit and Celsius, to figure floor area for calculating how much flooring to buy, or to calculate the area of the top of the piston for determining displacement of an engine. Electricians make frequent use of Ohm's law and the Pythagorean theorem. Carpenters use the Pythagorean theorem (3:4:5 triangle) in connection with stair and roof construction. Mechanics use formulas to calculate compression ratios, hydraulic pressures, and torque, among other things.

Key terms

Addition/subtraction principle: The principle stating that if the same number is added or subtracted to both sides of an equation, the solution remains unchanged. For example, if $x = a$ then $x + 3 = a + 3$.

Binomial: A polynomial consisting of two terms.

Brackets: Symbols, such as ( ) or [ ] or { }, used to enclose expressions.

Coefficient: The number part of an algebraic term; for example, –5 is the coefficient of the term $-5x$.

Common factor: A number and/or variable that divides evenly into each term of an expression.

Equation: A statement that says one expression is equal to another expression. For example, $2x + 3 = 9$ is an equation.

Expression: Numbers, symbols, and operators (such as + and ×) grouped together that show the value of something. An expression is a group of terms separated by + or – signs; for example, $2x + 3$ is an expression.

Factor a polynomial: The term used to describe writing a polynomial as a product.

Formula: An equation that contains two or more variables that represent unknowns. Formulas state how one quantity is related to other quantities; for example, P = 2L + 2W is a formula.

Like terms: Terms that have exactly the same variables including variable exponents; for example, $12x2$ and $5x2$ are like terms, but x^3 and x^2 are not like terms.

Monomial: A polynomial consisting of only one term.

Multiplication/division principle: The principle stating that if the same number is multiplied or divided to each side of an equation, the solution remains unchanged. For example, if $x = a$ then $3x = 3a$.

Polynomial: A many-termed algebraic expression; for example, $x2 + 2x - 5$ is a polynomial.

Sign: When it is used on its own by just saying "sign," it means negative or positive, such as "what sign is the number, positive or negative?"

Solution: Any variable replacement that makes an equation a true statement.

Term: A number or a number multiplied by one or more variables; for example, $5x2y$ is a term.

Trinomial: A polynomial consisting of three terms.

Variable: A symbol, usually a letter, used to represent a value in an equation. For example, the variable in the equation $2x + 3 = 9$ is the letter x.

Solving equations

An equation is a statement that says one expression is equal to another expression. The equation

$$2x - 5 = 7$$

states that the expression $2x - 5$ is equal to the term 7. The x in the equation $2x - 5 = 7$ is called a variable. Initially the variable is an unknown.

To solve an equation means to find a number for the variable that makes the equation a true statement. For example, 6 is a solution to the equation $2x - 5 = 7$, because when $x = 6$:

$$2x - 5 = 7$$
$$2(6) - 5 = 7$$
$$12 - 5 = 7$$
$$7 = 7$$

The statement $2(6) - 5 = 7$ is a true statement. Note that there is no other number that is a solution to $2x - 5 = 7$.

If $x = 10$, then $2(10) - 5 = 7$ is a false statement and 10 is not a solution to the equation $2x - 5 = 7$.

Some equations have no solution. For example, $x = x + 1$ has no solution. (No number can be equal to one more than itself.) There is no replacement for x that makes $x = x + 1$ a true statement.

Some equations have an infinite number of solutions. For example, $2x = x + x$ has an infinite number of solutions. Any replacement for x makes $2x = x + x$ a true statement. Try it.

The above kinds of equations are rare. Most of the equations we will deal with in this Learning Task have only one solution.

Multiplication and division principles

The equation

$5x = 30$

states that 5 times some number is 30. Since we know that 5 times 6 is 30, the solution to the equation, $5x = 30$, is $x = 6$.

The equation

$34x = 238$

is not as simple to solve. Few of us know what number times 34 is 238. However, we can solve this equation algebraically using the division principle.

The division principle states that if we divide both sides of the equation by the same number, the solution to the equation will remain unchanged.

To solve the above equation means to find x or $1x$. And, to find $1x$, divide both sides of the equation by the coefficient of x term.

To solve $34x = 238$ algebraically, write:

$34x = 238$

$\frac{34x}{34} = \frac{238}{34}$ divide both sides by 34

$1x = 7$ note that $\frac{34x}{34} = 1x$ and $\frac{238}{34} = 7$

Study the following examples.

Example 1
Solve $5x = 36$.

Solution

$5x = 36$

$\frac{5x}{5} = \frac{36}{5}$ divide both sides by 5

$x = 7\frac{1}{5}$ or $x = 7.2$

Check: $5(7.2) = 36$, so the statement is true.

Example 2

Solve $13 = -\frac{1}{2}x$.

Solution

$$13 = -\frac{1}{2}x$$

$$\frac{13}{-\frac{1}{2}} = \frac{-\frac{1}{2}x}{-\frac{1}{2}}$$

$-26 = 1x$ or $x = -26$

Check $13 = -\frac{1}{2}(-26)$, so the statement is true.

Likewise, you can multiply both sides of an equation by the same number without affecting the solution of the equation.

Example 3

Use the multiplication principle to solve $13 = -\frac{1}{2}x$.

Solution

$$13 = -\frac{1}{2}x$$

$-2(13) = 2(-\frac{1}{2}x)$ multiply both sides of the equation by -2

$-26 = 1x$ or $= -26$ note that $-2(-\frac{1}{2}x) = 1x$

Check: $13 = -\frac{1}{2}(-26)$, so the statement is true.

This is just the beginning. We will use various algebraic principles to solve more complicated equations. However, each time we will go through a series of steps that eventually leads to the statement $1x =$ the solution.

Addition and subtraction principles

Equations like

$$3x - 7 = 8$$

can be solved using two principles. If the same number is added to both sides of a true equation, the equation remains true.

Step 1

When solving an equation like

$$3x - 7 = 8,$$

the goal is to get the variable term isolated on one side of the equation and the number term on the other. If you add 7 to both sides of the above equation, you will have

$$3x - 7 + 7 = 8 + 7$$

and

$$3x = 15$$

The addition principle states that if the same number is added to both sides of a true equation, the equation remains true.

Step 2

To find x, now divide by 3, using the division principle.

$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

Another way of isolating the variable term is to move the number term to the other side and change its sign. For example:

$$3x - 7 = 8$$
$$3x = 8 + 7$$

When a term changes sides, it also changes signs.

Example 1

Solve $4x - 3 = 47$.

Solution

To isolate the $4x$ term, move the -3 term to the other side and change its sign.

$$4x - 3 = 47$$
$$4x = 47 + 3$$
$$4x = 50$$
$$\frac{4x}{4} = \frac{50}{4}$$
$$x = 12.5$$

Check this solution in the equation above.

Example 2
Solve $15 - 6x = 12$,

Solution
Move the 15 to the other side and change its sign.

$$15 - 6x = 12$$
$$-6x = 12 - 15$$
$$-6x = -3$$
$$\frac{-6x}{-6} = \frac{-3}{-6}$$
$$x = \frac{1}{2} \text{ or } 0.5$$

Check this solution in the equation above.

Example 3
Solve $-4.5 = 0.2x - 7.5$,

Solution
Move the -7.5 to the other side and change its sign.

$$-4.5 = 0.2x - 7.5$$
$$7.5 - 4.5 = 0.2x$$
$$3 = 0.2x$$
$$\frac{3}{0.2} = \frac{0.2x}{0.2}$$
$$15 = x$$

Check this solution in the equation above.

Many-termed equations

We have already seen that when a term changes sides, it also changes signs. We can extend this idea to changing variable terms from one side of an equation to another side.

Example 1

Solve $5x = 16 - 3x$

Solution

The strategy is to get all the *x*-terms on the same side and have the number term on the other side.

Move the $-3x$ to the other side and change its sign.

$$5x = 16 - 3x$$
$$3x + 5x = 16$$
$$8x = 16$$
$$\frac{8x}{8} = \frac{16}{8}$$
$$x = 2$$

Example 2

Solve $14x - 8 = 9x + 10$

Solution

Here, move the *x*-terms to one side and the number terms to the other.

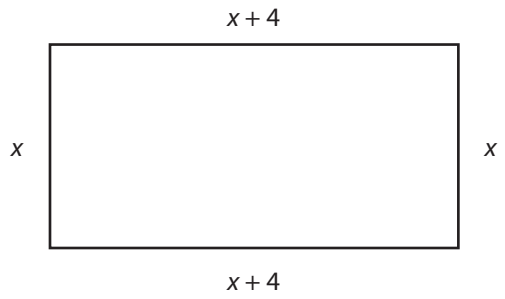
$$14x - 8 = 9x + 10$$
$$14x - 9x = 10 + 8$$
$$5x = 18$$
$$\frac{5x}{5} = \frac{18}{5}$$
$$x = 3\frac{3}{5} \text{ or } 3.6$$

Example 3

The length of a rectangle is 4 cm more than the width of the rectangle. If the perimeter of the rectangle is 100 cm, what are the dimensions of the rectangle?

Solution

Draw a diagram of the rectangle and label its sides. Note that if the width is *x* cm, the length is 4 more cm, or $x + 4$.



The equation is

$$x + x + 4 + x + x + 4 = 100$$
$$4x + 8 = 100$$
$$4x = 100 - 8$$
$$4x = 92$$
$$\frac{4x}{4} = \frac{92}{4}$$
$$x = 23$$

The width is x or 23 cm.

The length is $x + 4$ or $23 + 4 = 27$ cm.

Note that the question asked for the dimensions of the rectangle, so the answer must give both the length and width.

Equations with brackets

There is another type of equation that involves parentheses (or brackets). For example, the equation

$$3 + 2(x - 5) = 29$$

has brackets. Before you can solve this equation, you must simplify it to remove the brackets. Then you can use the principles of addition and subtraction to solve it.

Example 1
Remove brackets from $2(3 + 8x)$.

Solution
Multiply both the 3 and the $8x$ by 2.

$$2(3 + 8x)$$
$$= 2(3) + 2(8x)$$
$$= 6 + 16x$$

Example 2
Remove brackets from $-(x-4)$.

Solution
There doesn't appear to be a number in front of the brackets, but there is. It is –1. Multiply both x or $1x$ and 4 by –1.

$$\begin{aligned}
&-(x-4)\\
&=-1(x-4)\\
&=-1(1x)-(-1)(4)\\
&=-1x-(-4)\\
&=-x+4
\end{aligned}$$

Note in the above example that a negative sign in front of the brackets has the effect of changing the inside terms to their opposites. The shortcut is:

$$-(x-4)=-x+4$$

Example 3
Simplify the expression $2-5(2x-7)$. Recall the order of operations, BEDMAS.

Solution
Remove the brackets first, but do not subtract $2-5$. Multiply both $2x$ and 7 by –5. Be careful, and notice that –(–35) means +35. Add $2+35$.

$$\begin{aligned}
&2-5(2x-7)\\
&=2-10x-(-35)\\
&=2-10x+35\\
&=-10x+37
\end{aligned}$$

Formulas

Formulas are equations that contain two or more variables that represent unknown values. If you know the values of all but one of the variables, you can substitute these values into the formula and solve the formula for the unknown variable.

Example 1
You can use the formula A = LW to find the area (A) of a rectangle if you know the length (L) and the width (W). Find the area of a rectangle with a length of 8 cm and a width of 5 cm.

Solution

$$\begin{aligned}
A &= LW\\
A &= 8\text{ cm}\times 5\text{ cm}\\
A &= 40\text{ cm}^2
\end{aligned}$$

Example 2
Use the formula $P = 2L + 2W$ to find the perimeter of the above rectangle.

Solution

$P = 2L + 2W$
$P = 2(8\text{ cm}) + 2(5\text{ cm})$
$P = 16\text{ cm} + 10\text{ cm}$
$P = 26\text{ cm}$

Example 3
The formula $C = \frac{5}{9}(F - 32)$ is used to find Celsius temperature given Fahrenheit temperature. Find the Celsius temperature that corresponds to 104° Fahrenheit.

Solution

$$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}(104 - 32)$$

$$C = \frac{5}{9}(72)$$

C = 40°, so 104°F is equal to 40°C.

Formula manipulation (transposing)

It is often useful to rearrange a formula to solve for an unknown variable. To do this, isolate the required letter on one side of the equals sign while all other symbols are on the opposite side. Remember that formulas are equations. Use the same principles in solving that you use for any other equations.

Example 1
$A = P + I$ is a formula that gives the amount of money (A) an investor collects on a principal (P) when the interest is (I).

Suppose you know the amount collected (A) and the interest (I). You wish to calculate the principal (P).

Solution
To solve the formula for P, you need to get P alone on one side.

Solve for P

$$P + I = A$$

Subtract I from each side

$$P + I - I = A - I$$
$$P = A - I$$

Example 2

A = LW is the formula that gives the area (A) of a rectangle with length (L) and width (W).

Solution

Solve for W

$$A = LW$$

Divide both sides by L

$$\frac{A}{L} = \frac{LW}{L}$$

$$\frac{A}{L} = W \text{ or } W = \frac{A}{L}$$

Example 3

$A = \frac{1}{2}bh$ finds the measure of the area (A) of a triangle with base (*b*) and height (*h*).

Solution

Solve for *b*.

$$\frac{1}{2}bh = A$$

Multiply both sides by 2

$$2\left(\frac{1}{2}BH\right) = 2A$$

$$BH = 2A$$

Divide both sides by *h*

$$\frac{bh}{h} = \frac{2A}{h}$$

$$B = \frac{2A}{H}$$

Example 4

$F = \frac{9}{5}C + 32$ is a means of finding the temperature in degrees Fahrenheit (F) when the temperature in degrees Celsius (C) is known.

Solution

Solve for C

$$\frac{9}{5}C + 32 = F$$

Subtract 32 from both sides

$$\frac{9}{5}C + 32 - 32 = F - 32$$

$$\frac{9}{5}C = F - 32$$

Multiply through by 5

$$5 \times \frac{9}{5}C = 5(F - 32)$$

$$9C = 5(F - 32)$$

Divide through by 9

$$\frac{9C}{9} = \frac{5(F - 32)}{9}$$

$$C = \frac{5(F - 32)}{9}$$

Example 5

$A = \pi r^2$ is the formula for the area (A) of a circle where r is the radius and π is a constant.

Solution

Solve for r

$$\pi r^2 = A$$

Divide both sides by r

$$\frac{\pi^2}{\pi} = \frac{A}{\pi}$$

$$r^2 = \frac{A}{\pi}$$

Take the square root of both sides

$$\sqrt{r^2} = \sqrt{\frac{A}{\pi}}$$

$$r = \sqrt{\frac{A}{\pi}}$$

Some hints for solving a formula:

1. **To solve a formula for a given variable you must isolate the variable on one side of the equals sign.**
2. **To isolate a variable, remove all other terms from the same side. Do this by performing the inverse operation:**
   - **division is the inverse of multiplication.**
   - **addition is the inverse of subtraction.**
   - **square rooting $\left(\sqrt{x}\right)$ is the inverse of squaring (x^2).**
3. **Remember that formulas are equations. Whatever operations you perform on one side of the equals sign must also be performed on the other side.**

Now complete the Learning Task Self-Test.

Self-Test 6

1. $11 = n - 3$
   a. $n = 7$
   b. $n = 14$
   c. $n = 21$
   d. $n = 10$

2. $27 = 8(n + 3)$
   a. $n = 2$
   b. $n = 3/4$
   c. $n = 3/8$
   d. $n = 3/16$

3. $x2 + 4 = 20$
   a. 4
   b. 8
   c. 16
   d. 24

4. $C = \pi 42$
   a. C = 133.648
   b. C = 131.946
   c. C = 131.947
   d. C = 131.946

5. $2[13 + (8 - 5)] = x$
   a. $x = 52$
   b. $x = 32$
   c. $x = 23$
   d. $x = 43$

6. $N + 2(3 - 1) = 8$
   a. N = 2
   b. N = 0
   c. N = 4
   d. N = 6

7. 3N = 123
   a. N = 14
   b. N = 41
   c. N = 36
   d. N = 63

8. 15 = 29 – (2*x*)
   a. x = 7
   b. x = 5
   c. x = 14
   d. x = –7

9. Use the formula A = LW, where

   A = area
   L = length
   W = width

   Solve for the missing dimensions.

RECTANGLE	LENGTH	WIDTH	AREA
A	250 cm	150 cm	_____cm^2
B	7.5 cm	_____cm	18 cm^2
C	_____cm	475 cm	570 000 cm^2
D	28 cm	_____cm	644 cm^2

10. Use the formula P = EI, where

    P = power in watts (w)
    E = force in volts (v)
    I = current in amperes (A)

    Calculate the power in watts for an electric clothes dryer circuit that has 24 A and 220 V.

    a. P = 9.16 W
    b. P = 1200 W
    c. P = 2640 W
    d. P = 5280 W

11. Use the formula for calculating the taper of a shaft $t = \frac{D - d}{L}$, where

    t = taper in centimetres per metre (cm/m)
    D = diameter of the larger end
    d = diameter of the smaller end
    L = length in metres

    Determine the taper of a shaft that has end diameters of 5.3 cm and 3.1 cm and a length of 1.3 metres.

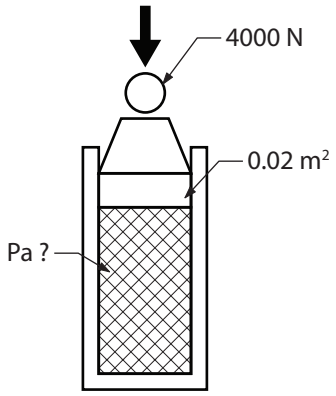
    a. t = 1.7 cm/m
    b. t = 1.4 cm/m
    c. t = 1.2 cm/m
    d. t = 1.1 cm/m

12. Use the formula F = PA, where

    F = force in newtons (N)
    P = pressure in pascals (Pa)
    A = area in square metres (m^2)

    Calculate the pressure in the hydraulic system illustrated below:

    a. P = 80 Pa
    b. P = 8000 Pa
    c. P = 20 000 Pa
    d. P = 200 000 Pa



13. Use the conversion formula $F = \frac{9}{5}(C + 32)$, where

    F = degrees Farhenheit
    C = degrees Celsius

    Convert 175°C to degrees Farhenheit.

    a. F = 337°F

    b. F = 372.6°F

    c. F = 378.8°F

    d. F = 388.8°F

14. Use the formula $A = \pi r^2$, where

    A = area of a circle
    r = radius of the circle

    Determine the radius in centimetres of a circle with an area of 255 cm^2.

    a. 9 cm

    b. 18 cm

    c. 44 cm

    d. 81.169 cm

LEARNING TASK 7

Solve problems involving perimeters, areas, and volumes

Working with pipe and conduit requires knowledge of the area and volume of circles and cylinders, as does many aspects of engine work in the mechanics trades. Virtually all parts of any trade require working with these dimensions. The precise dimensions required for detail and layout work in all trades are equally dependent on careful calculation.

Key terms

Altitude: Also referred to as *height*. The perpendicular distance between the base of a triangle or other figure and its uppermost side or point.

Area: The amount of surface enclosed by a figure.

Base: The lower side of a triangle or other figure.

Circumference: The distance around a circle.

Composite: Something made up of several simpler parts.

Diameter: The line segment that joins two points on a circle and passes through the centre of the circle.

Formula: A shortcut method of finding an unknown numerical quantity when other quantities are known; for example, a formula to find the area of a rectangle:

$$\text{area} = \text{length} \times \text{width}, \text{ or } A = L \times W$$

Height: See Altitude.

Perimeter: The distance around a figure.

Perpendicular: A term used to mean "at right angles." Two lines are perpendicular when the angle between them is a right angle, or has a measure of 90°.

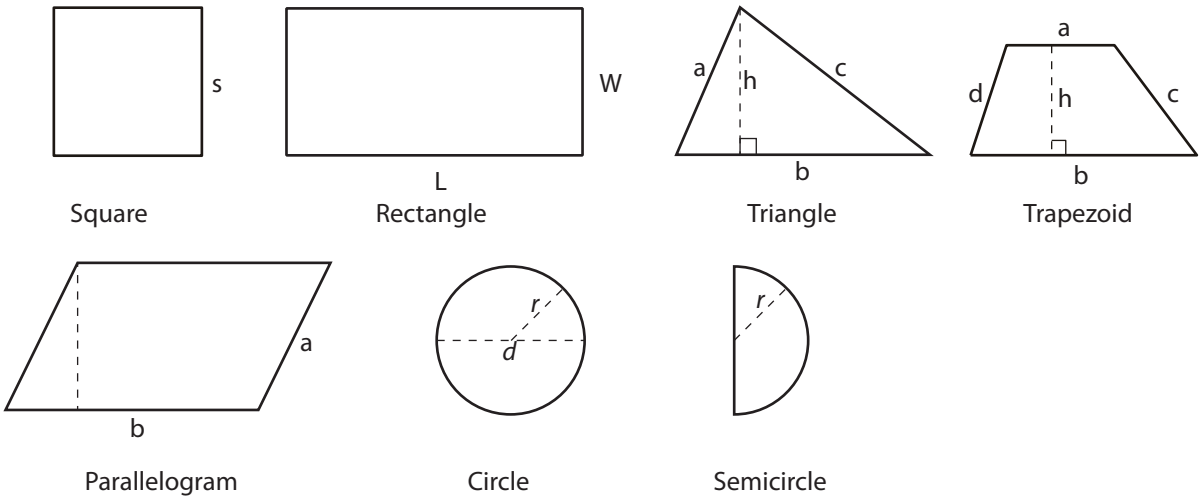
Pi (π): A Greek letter, which stands for the irrational number that begins 3.14159265...

Quadrilateral: A four-sided figure.

Radius: The line segment from the centre of a circle to any point on the circle. The length of a radius is one-half the length of a diameter.

Volume: The amount of space occupied by a solid.

Figures

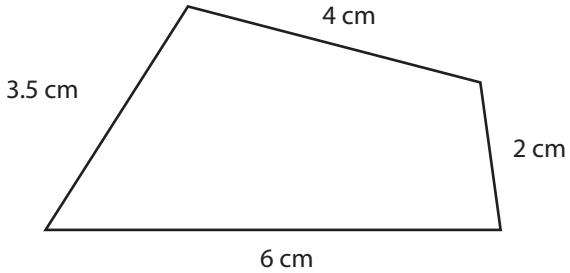


Perimeter

The perimeter of a figure is the total distance around the figure.

Example

Find the perimeter of the quadrilateral shown. (A quadrilateral is a four-sided figure.)



Solution

Measure and label each side of the figure. Add the four sides.

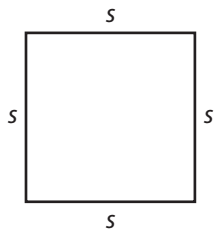
P = 4 cm + 2 cm + 6 cm + 3.5 cm = 15.5 cm
(The letter P is the symbol for perimeter.)

Perimeter formulas

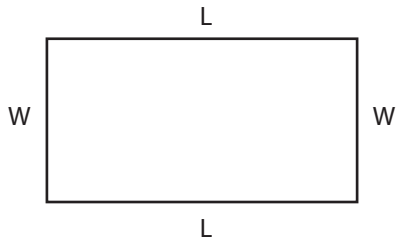
Finding the perimeter of a figure by measuring all of its sides can sometimes be time consuming. For certain figures, such as squares, rectangles, and circles, you can use perimeter formulas that shortcut the measuring process. For example, the perimeter of a square can be found by measuring one side and multiplying by four. The formula is:

Perimeter = 4 × side
P = 4*s*

(Recall that "4s" means "4 times *s*")

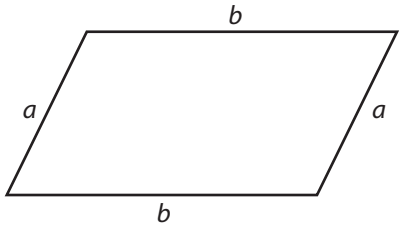


Study the following.



Perimeter = 2× (length + width)

P = 2(L + W)



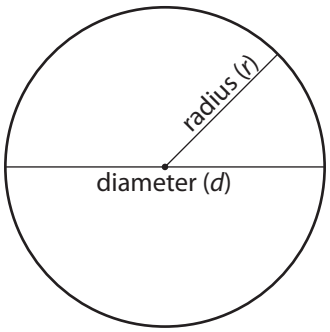
Perimeter = 2× (side a + side b)

P = 2(a + b)

Circles

The diameter of a circle is the distance from one side of the circle through the centre to the other side. The radius is the distance from the centre of the circle to the edge of the circle. The radius is one half the diameter or:

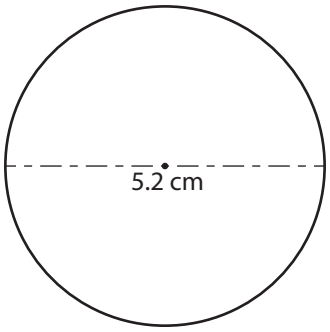
$d = 2r$
Perimeter = pi × diameter, or
Perimeter = 2 × pi × radius
$P = \pi d$ or $P = 2\pi r$



The number pi, or π, is impossible to state exactly. A good approximation for π is 3.1416. The number π can only be approximated. It is one of many irrational numbers. Most scientific calculators have a π button. π is the Greek letter pronounced "pi." $\pi = 3.14159265...$

The perimeter of a circle is also referred to as the *circumference* of the circle.

Example 1
Find the circumference of the circle. Round to one decimal place.

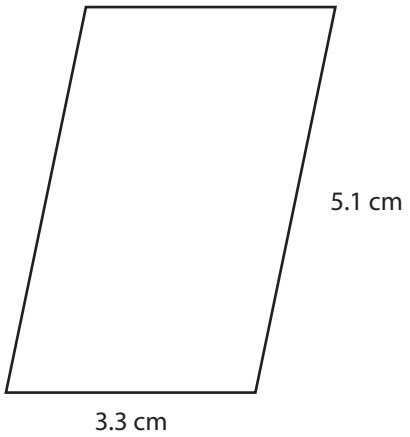


Solution
Measure the diameter of the circle. Use the formula:

$P = \pi d$
$P = 3.1416 \times 5.2$
$P = 16.33632 \text{ cm} \approx 16.3 \text{ cm}$

The symbol ≈ means "approximately equal to."

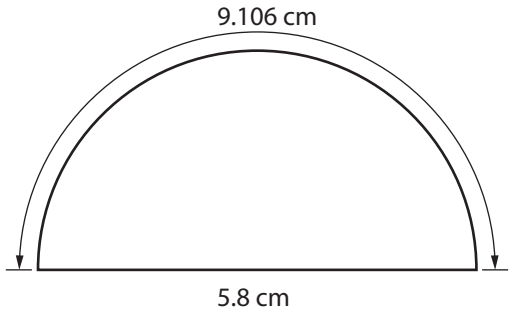
Example 2
Find the perimeter of the parallelogram shown.



Solution
Measure two adjacent sides. Use the formula:

$P = 2(a + b)$
$P = 2\ (3.3 \text{ cm} + 5.1 \text{ cm})$
$P = 2(8.4 \text{ cm})$
$P = 16.8 \text{ cm}$

Example 3
Find the perimeter of the semicircle shown. Round to one decimal place.



Solution
Measure the diameter. Find the perimeter of the half circle.

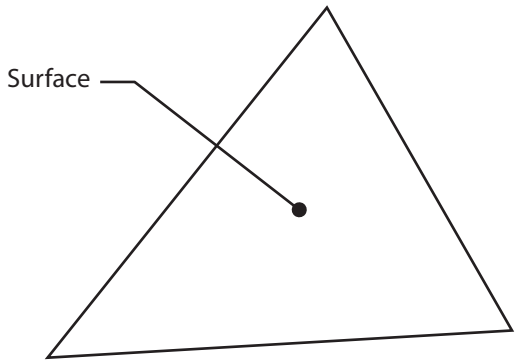
$P = \pi d \div 2$
$P = 3.1416 \times 5.8 \text{ cm} \div 2$
$P = 18.22128 \text{ cm} \div 2$
$P = 9.11064 \text{ cm}$

Now add the diameter length and round the answer to one decimal place.

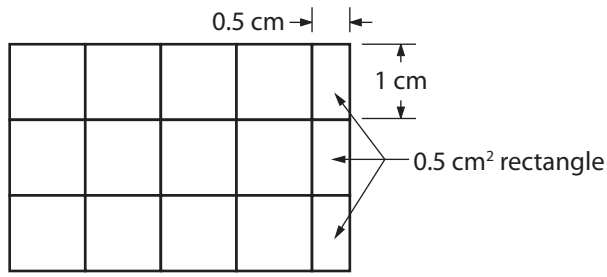
$9.11064 + 5.8 \text{ cm} = 14.91064 \approx 14.9 \text{ cm}$

Area

The area of a figure is the amount of surface enclosed by the figure.



The area of a figure can be determined by counting the number of squares (and parts of squares) that are needed to cover the particular surface. For example, in the rectangle shown below, 13.5 square centimetres (count them) are needed to cover the surface of the rectangle.

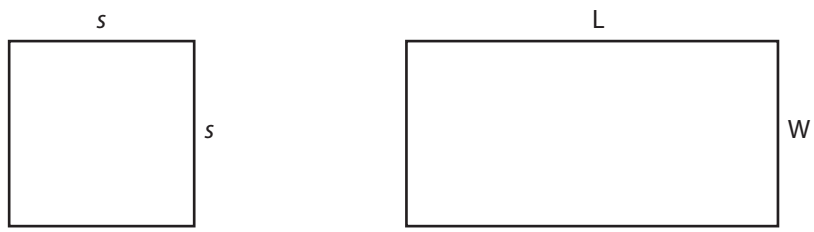


We say that the area of this rectangle is 13.5 cm^2.

The common metric units of area are km^2, m^2, and cm^2.

Area formulas

Counting squares and parts of squares is a fairly tedious way of determining the area of a figure. Fortunately, there are simple area formulas for the more common geometrical figures.



Area = side × side

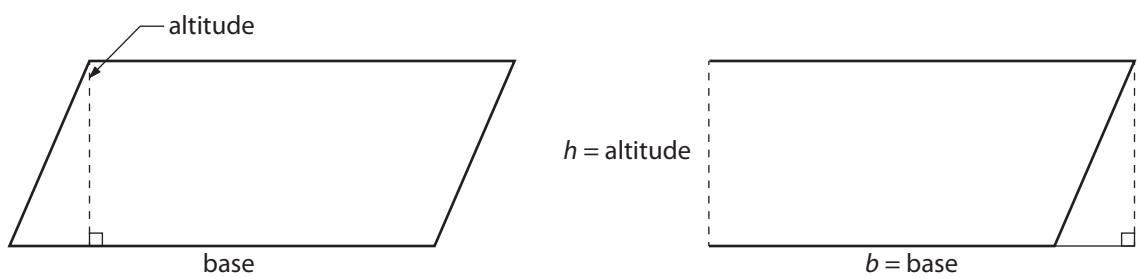
$A = s^2$

Area = length × width

$A = LW$

Note: s^2 means $s \times s$ and s^2 is read as "*s* squared."

Knowing the area of a rectangle enables us to find the area of parallelograms, trapezoids, and triangles. Notice how the parallelogram below has been transformed into a rectangle with exactly the same area. The corner triangle can be "cut off" and shifted to the far side of the parallelogram.

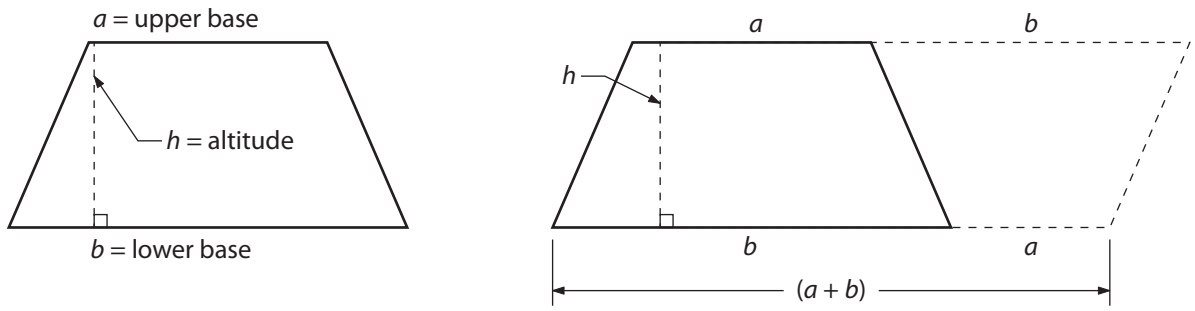


Area = base × altitude

$A = bh$

To find the area of a parallelogram you need only two measurements: the base and the altitude. The altitude or height is the perpendicular distance between the lower and upper parallel sides of the parallelogram.

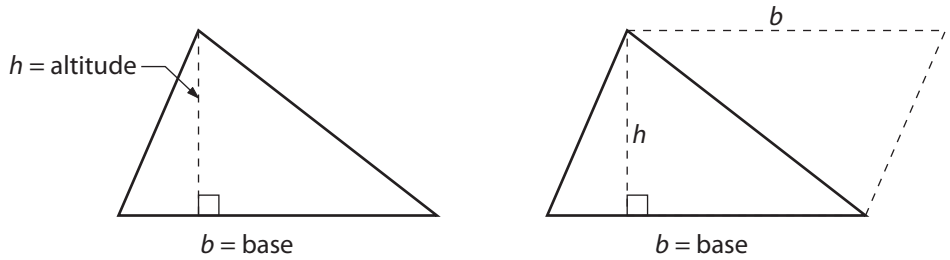
Consider the trapezoid below. This time, imagine that you trace out the original trapezoid, flip it upside down, and slide it over to the far end. Now you have created a parallelogram that has twice the area of the original trapezoid. Or, the trapezoid is one-half the area of the larger parallelogram.



Area = ½ × altitude × (lower base + upper base)

$A = \frac{1}{2}h(a + b)$

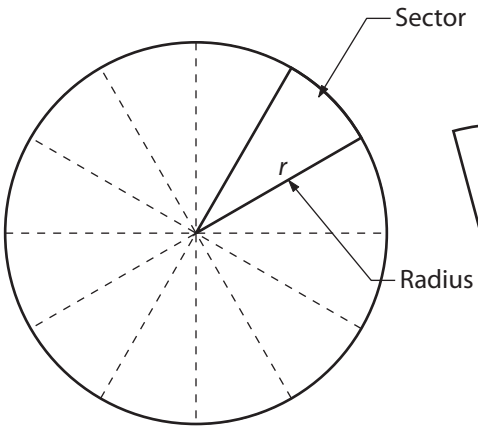
In the triangle below, you can again trace another identical triangle and flip it about one of the triangle's sides to, once again, obtain a parallelogram with an area twice that of the original triangle. So, the area of the triangle is one-half the area of the parallelogram.



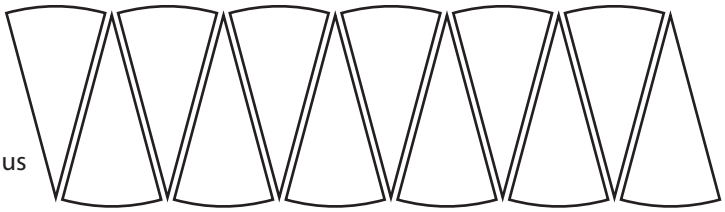
Area = ½ × base × altitude

$A = \frac{1}{2}bh$

Now, consider the circle below. Imagine slicing it into thin pieces called *sectors*. Now form a row of sectors. The thinner the slices or sectors are, the more this row will form the figure of a rectangle with a width equal to the radius of the circle and a length equal to one-half the circumference (or perimeter) of the circle.



$\frac{1}{2}$ P = πr



Area = π × radius × (or π × radius squared)

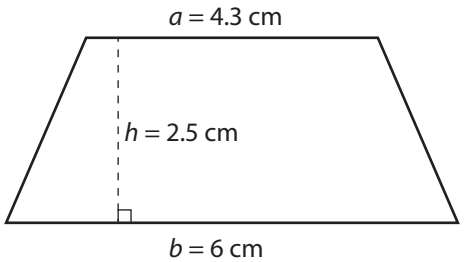
$A = \pi r^2$

Examples using area formulas

Study the following examples.

Example 1

Find the area of the trapezoid.



Solution

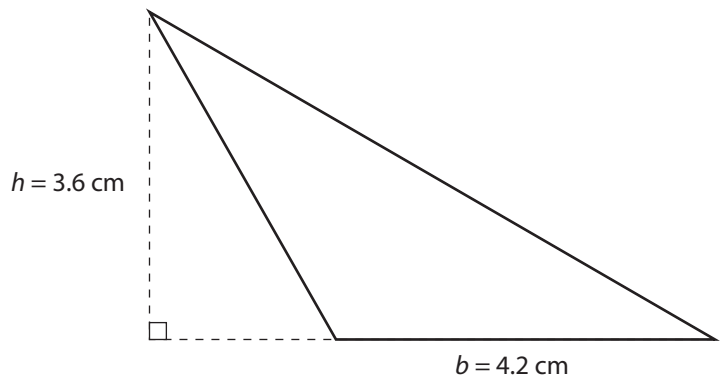
Measure the length of the upper base, lower base, and altitude. Label these lengths.

Apply the trapezoid area formula.

$A = \frac{1}{2}h(a + b)$
$A = \frac{1}{2}(2.5\text{ cm})(4.3\text{ cm} + 6\text{ cm})$
$A = 0.5(2.5\text{ cm})(10.3\text{ cm})$
$A = 12.875\text{ cm}^2 \approx 12.9\text{ cm}^2$

Example 2

Find the area of the triangle.

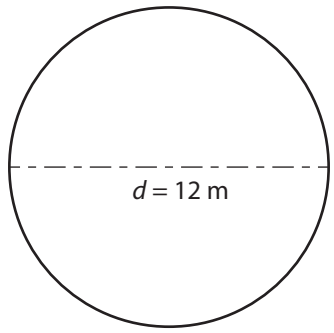


Solution
Simply measure and label the base and altitude. (Note that the altitude must make a 90-degree angle with the base, which in this case is extended.)

Use the area formula.

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(4.2\text{ cm})(3.6\text{ cm})$
$A = 0.5(15.12\text{ cm}^2)$
$A = 7.56\text{ cm}^2 \approx 7.6\text{ cm}^2$

Example 3
Find the area of a circle with a diameter of 12 m.



Solution
The area formula for the circle involves only the radius. To find the radius,

$r = 12\text{ m} \div 2 = 6\text{ m}$

To find the area,

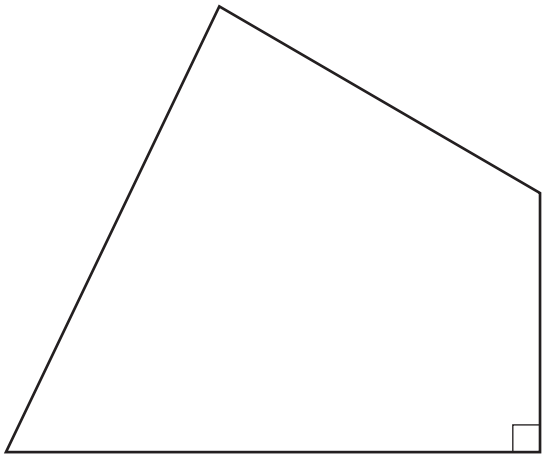
$A = \pi r^2 = 3.1416(6\text{ m})^2$
$A = 3.1416(36\text{ m}^2)$
$A = 113.0976\text{ m}^2 \approx 113.1\text{ m}^2$

Areas of irregular figures

An irregular figure can be thought of as a combination of familiar figures. To determine the area of an irregular figure, either divide the figure into several familiar areas and find their sum or subtract one area from another. Study the examples below.

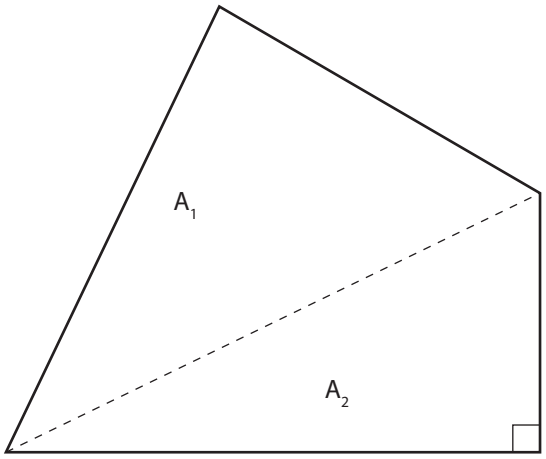
Example 1

Find the area of the quadrilateral.

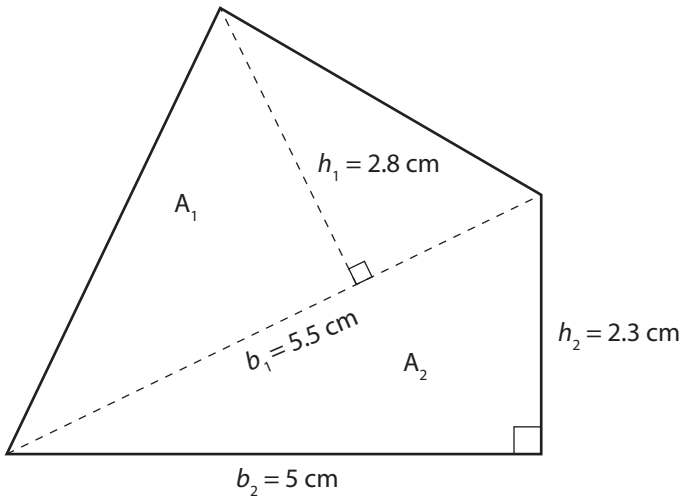


Solution

Step 1: Divide the quadrilateral into two triangles, A_1 and A_2, as shown. Use a 90 degree right angle to construct an altitude for the upper triangle. Note that one side of the lower right triangle is also an altitude.



Step 2: Measure and label the bases and their altitudes.
These are the only measurements needed.



Step 3: Now determine the area of both triangles, A_1 and A_2, and then find their sum, $A_1 + A_2$.

$A_1 = ½b_1h_1$
$A_1 = ½(5.5 \text{ cm})(2.8 \text{ cm})$
$A_1 = 0.5(15.4 \text{ cm}^2) = 7.7 \text{ cm}^2$

$A_2 = ½b_2h_2$
$A_2 = ½(5 \text{ cm})(2.3 \text{ cm})$
$A_2 = 0.5(11.5 \text{ cm}) = 5.75 \text{ cm}^2$

$A_1 + A_2 = 7.7 \text{ cm}^2 + 5.75 \text{ cm}^2 = 13.45 \text{ cm}^2 \approx 13.5 \text{ cm}^2$

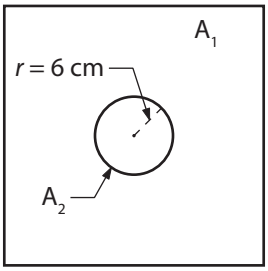
As seen in the example above, there are three important steps in finding the area of an irregular figure:

Step 1: Divide the figure into several familiar areas.

Step 2: Make only those measurements necessary to find the familiar areas.

Step 3: Calculate the areas and add or subtract as necessary.

Example 2
Find the area of the square washer with a hole.



s = 24 cm

Solution

Step 1: Let A_1 be the area of the square and A_2 be the area of the circle. Subtract $A_1 - A_2$ to determine the area of the washer.

Step 2: The necessary measurements are given.

Step 3: Determine A_1 and A_2 and then calculate $A_1 - A_2$ to find the area.

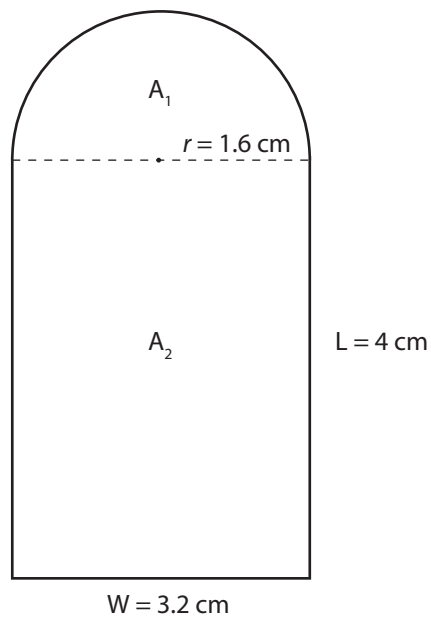
$A_1 = s^2$
$A_1 = (24\text{ cm})^2$
$A_1 = 576\text{ cm}^2$

$A_2 = \pi r^2$
$A_2 = 3.1416(6\text{ cm})^2 \approx 113.0976\text{ cm}^2$

$A_1 - A_2 = 576\text{ cm}^2 - 113.0976\text{ cm}^2 = 462.9024\text{ cm}^2 \approx 462.9\text{ cm}^2$

Example 3

Find the area of the figure shown.



Solution

Step 1: Divide the figure into the semicircle A_1 and the rectangle A_2.

Step 2: Measure only the radius of A_1 and the length and width of A_2.

Step 3: Calculate the areas.

$A_1 = \frac{1}{2}\pi r^2$
$A_1 = \frac{1}{2}(3.1416)(1.6\text{ cm})^2$

$A_1 = ½(3.1416)(2.56\ cm^2)$
$A_1 = 4.021248\ cm^2$

$A_2 = LW = 4.0\ cm\ (3.2\ cm)$
$A_2 = 12.8\ cm^2$

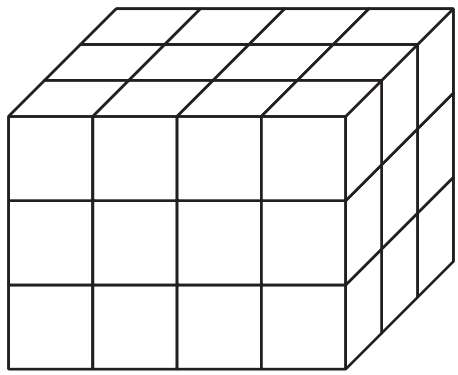
$A_1 + A_2 = 4.021248\ cm^2 + 12.8\ cm^2 = 16.821248\ cm^2 \approx 16.8\ cm^2$

Volume

The perimeter of an object is simply the total distance around that object. You can measure that distance with a ruler (a flexible ruler in the case of curved distance), and the units you use are mm, cm, m, or km.

Area measures the amount of surface taken up by a figure. You can measure a figure's area by counting the number of squares it takes to cover the figure. Fortunately, the area formulas can be used in place of actually counting squares. Area is a two-dimensional idea, hence the use of "2" in the area units of measurement: cm^2, m^2, and km^2.

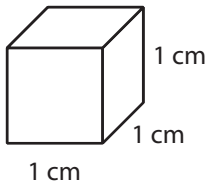
Volume measures the amount of three-dimensional space an object takes up. You can measure the volume of an object, or solid, by counting the number of cubes that fit inside that object. Can you determine how many cubes there are inside the rectangular prism below?



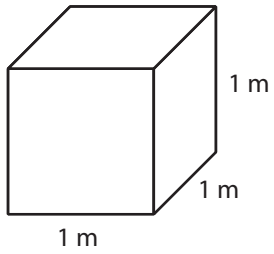
Did you "count" 36 cubes? Did you also notice that in the prism, L = 4, W = 3, and H = 3, and LWH = 4(3)(3) = 36?

The basic units of volume measurement are the:

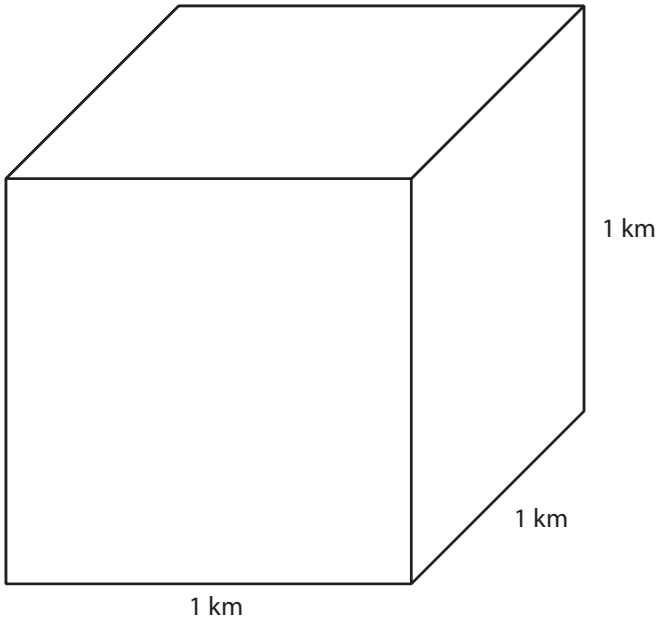
- cubic centimetre (cm^3)
- cubic kilometre (km^3)
- cubic metre (m^3)



cubic centimetre (cm^3)



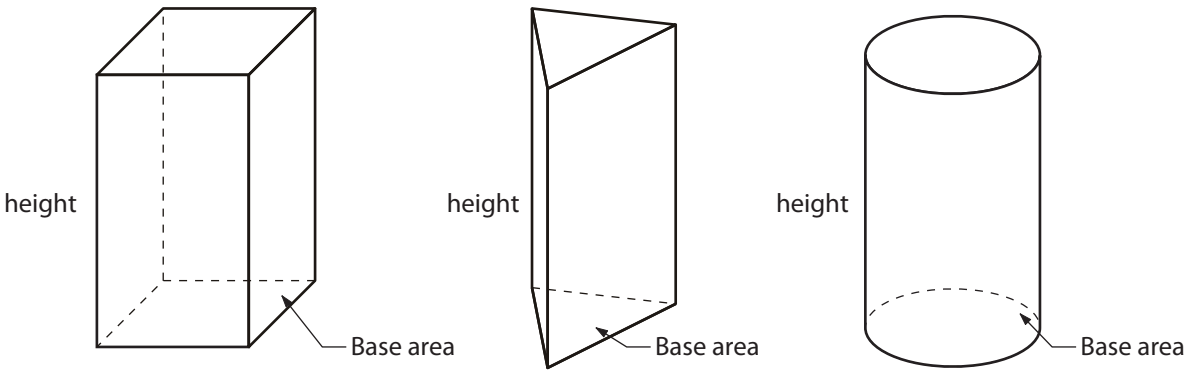
cubic metre (m^3)



cubic kilometre (km^3)

Fortunately, you do not have to go through the tedious activity of counting the number of cubes (and parts of cubes) that fit inside a regular solid object. Once again, you can apply some fairly simple volume formulas to determine the volume of a regular solid.

Although we will not spend a lot of time rigorously developing volume formulas for the following solids, you will probably notice a common idea running through each formula. The idea is that the volume of a regular solid can be determined by multiplying the base area of the solid by the solid's altitude or height. For example:

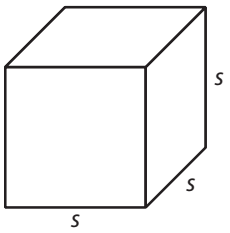


Volume = (base area) × (height)

$V = Ah$

Formulas for determining the volume of different solid shapes are as follows:

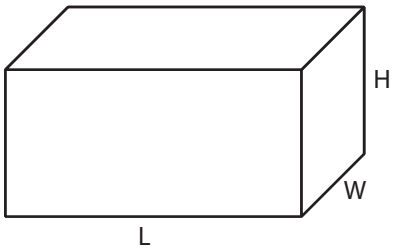
Cube



$V = s \times s \times s$ (read "*s* cubed")

$V = s^3$

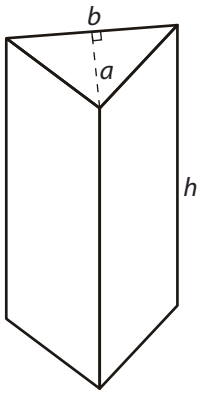
Rectangular prism



$V = L \cdot W \cdot H$

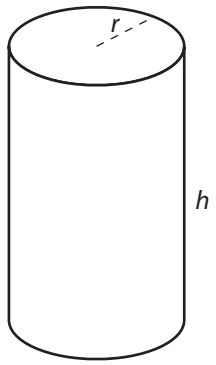
$V = LWH$

Triangular prism



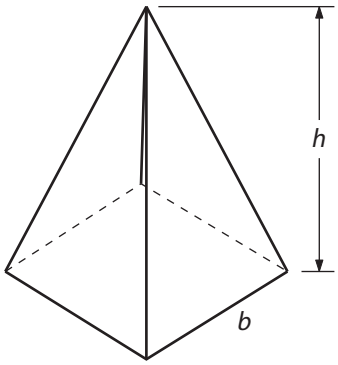
$V = \frac{1}{2}bah$

Cylinder



$V = \pi r^2 h$

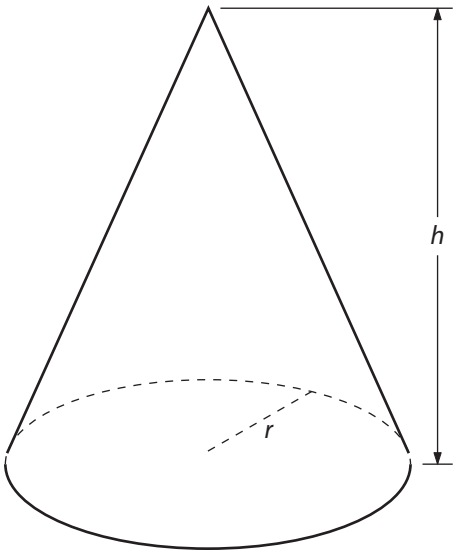
Pyramid



$V = \frac{1}{3} b^2 h$ or $V = b^2 h \div 3$

where *h* is the vertical height of the pyramid

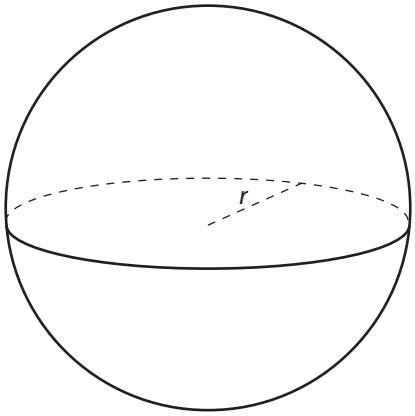
Cone



$V = \frac{1}{3} \pi r^2 h$ or $\pi r^2 h \div 3$

where *h* is the vertical height

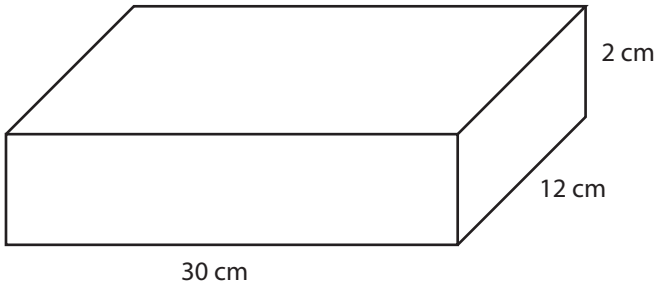
Sphere



$V = \frac{4}{3}\pi r^3$ or $V = 4\pi r^3 \div 3$

Example 1

Find the volume of the rectangular prism.



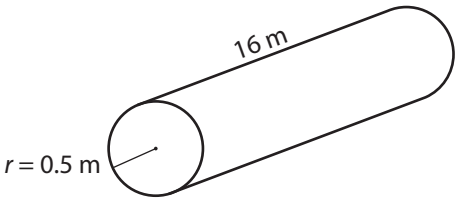
Solution

The measurements are given. Use the formula.

$V = LWH$
$V = (30\text{ cm})(12\text{ cm})(2\text{ cm})$
$V = 720\text{ cm}^3$

Example 2

Find the volume of a pipe whose radius is 0.5 m and height is 16 m.



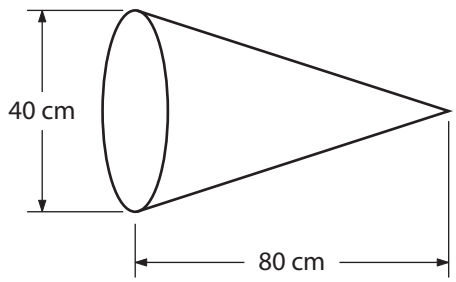
Solution

A pipe is a cylinder. Sketch and label it. Apply the formula.

$V = \pi r^2 h$

$V = 3.1416\ (0.5\ m)^2(16\ m)$
$V = 3.1416\ (0.25\ m^2)(16\ m)$
$V = 12.5664\ m^3$

Example 3
Find the volume of the cone.



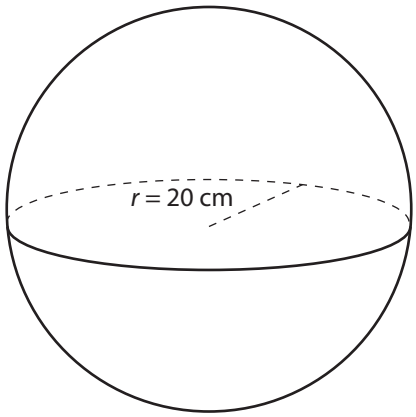
Solution
Note that the diameter of the base is given. Its radius is one-half this.

$$r = \frac{d}{2}$$
$$r = \frac{40\ cm}{2}$$
$$r = 20\ cm$$

Apply the formula.

$V = \pi r^2 h \div 3$
$V = (3.1416)(20\ cm)^2(80\ cm) \div 3$
$V = 33\ 510.4\ cm^3 \approx 33\ 500\ cm^3$

Example 4
Find the volume of a basketball with a radius of 20 cm.



Solution

Apply the volume formula.

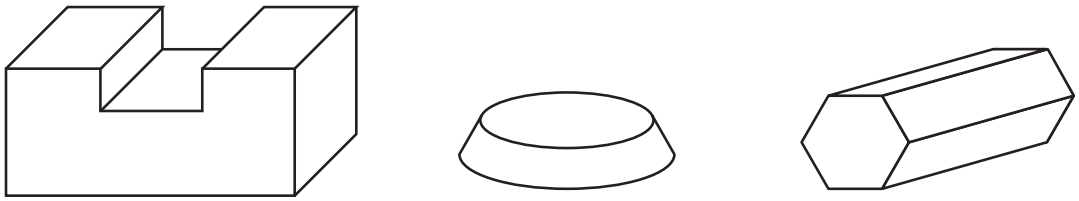
$V = 4\pi r^3 \div 3$

$V = 4(3.1416)(20\text{ cm})^3 \div 3$

$V = 33\,510.4\text{ cm}^3 \approx 33\,500\text{ cm}^3$

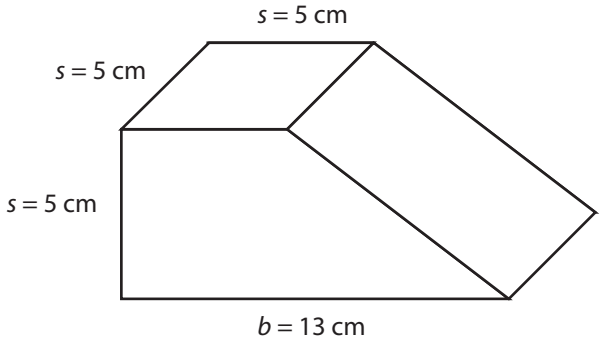
Volume of irregular solid figures

An irregular solid is a combination of or a portion of some regular solids. The solid figures below are irregular solids. Study the examples.



Example 1

Find the volume of the solid.



Solution

Divide the solid into the cube, V_1, and the triangular prism, V_2. Find V_1 and V_2.

$V_1 = s^3$

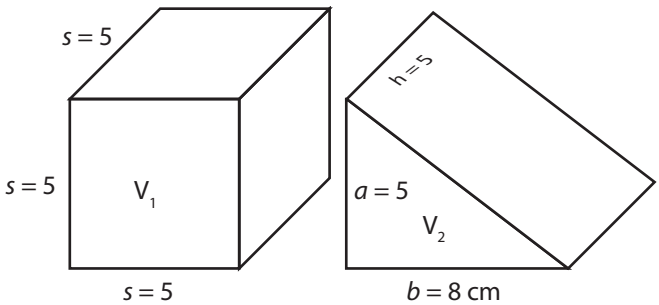
$V_1 = (5\text{ cm})^3$

$V_1 = 125\text{ cm}^3$

$V_2 = ½bah$

$V_2 = ½(8\text{ cm})(5\text{ cm})(5\text{ cm})$

$V_2 = 100\text{ cm}^3$

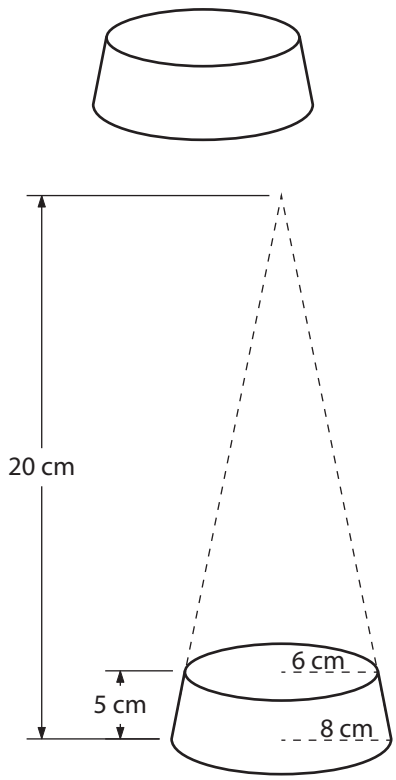


The total volume is

$V_1 + V_2 = 125\ cm^3 + 100\ cm^3 = 225\ cm^3$

Example 2

Find the volume of the support plate.



Solution

Consider the plate to be a portion of a cone. The volume of the large cone is:

$V_2 = \pi r^2 h \div 3$
$V_2 = 3.1416(8\ cm)^2(20\ cm) \div 3$
$V_2 = 1340.4\ cm^3$

The volume of the smaller "missing" cone is:

$V_1 = \pi r^2 h \div 3$
$V_1 = 3.1416(6\ cm)^2(20\ cm - 5\ cm) \div 3$
$V_1 = 565.488\ cm^3$

The volume of the support plate is:

$V_2 - V_1 = 1340.4\ cm^3 - 565.5\ cm^3 = 774.9\ cm^3 \approx 775\ cm^3$

Solving formulas with other variables

When you use formulas in practical applications, you may know the numbers for perimeter, area, or volume but be missing some other quantity. The examples below demonstrate how to solve formulas in such cases.

1. The area of a room is 36 m^2. If the room's width is 4.5 m, what is its length?
   - Insert the known quantities into the appropriate formula. A = LW
     36 = L(4.5)
   - Divide both sides of the equation by 4.5 to isolate the variable L.

     $$\frac{36}{4.5} = \frac{L(4.5)}{4.5}$$

     8 = L
   - Solve for L in metres.

     L = 8 m

2. A cylinder 15 cm tall holds 3000 cm^3 of liquid. What is the diameter of the cylinder?
   - Insert the known quantities into the appropriate formula. $V = \pi r^2 h$

     $3000 = 3.1416\,(r^2)15$
   - Divide both sides of the equation by 47.124 (3.1416 × 15) to isolate the variable r^2. Round the result to the precision of the approximate value of π.

     $$\frac{3000}{47.124} = \frac{47.124\,r^2}{47.124}$$

   - Take the square root of each side of the equation.

     $63.6618 = r^2$
   - Because diameter is two times the radius, multiply the radius 7.9788 by 2 to find the diameter D in centimetres.

     $$\sqrt{63.6618} = \sqrt{r^2}$$

     $$7.9788 = r$$

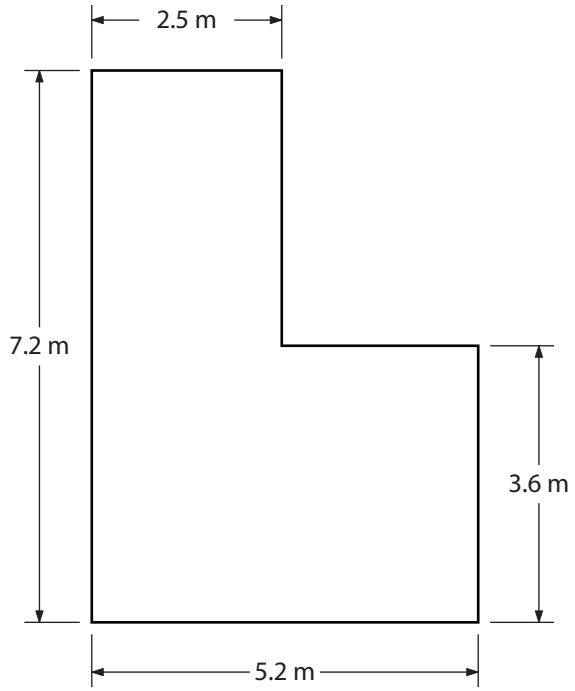
     $$D = 15.9576 \text{ cm}$$

Now complete the Learning Task Self-Test.

Self-Test 7

1. A construction site 27 m by 76 m must be fenced in before excavation starts. How many metres of fencing are required to enclose it?
   a. 2052 m
   b. 103 m
   c. 206 m
   d. 309 m

2. A contractor is deciding which floor covering to use in the room shown below. Cork floor tiles 0.25 m by 0.25 m cost $1.10 each, and carpeting costs $16.95 a square metre. Floor tiles would be cheaper than carpet.
   a. True
   b. False



3. A rectangular panel measuring 36" x 18" has two holes 6" in diameter to receive stereo speakers and a rectangular opening 8" x 10" for a rear window defroster. What is the surface area of the panel after the holes have been cut to the nearest square inch?
   a. 511 sq. in.
   b. 540 sq. in.
   c. 560 sq. in.
   d. 584 sq. in.

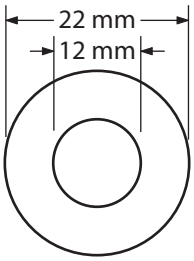
4. The cylindrical water tank servicing a construction site must be erected on a wooden cradle and fastened by three metal straps around the outside. If the tank is 44" in diameter, what is the total length of metal required for the straps, to the nearest 1⁄16"?
   a. 138¼"
   b. 276½"
   c. 452⁹⁄₁₆"
   d. 414¹¹⁄₁₆"

5. A pipe 35 cm in diameter must be inserted into a storage tank. What is the area of the hole that must be cut in the side of the tank if you leave 0.3 cm clearance on the outside of the pipe?
   a. 111.84 cm^2
   b. 995.38 cm^2
   c. 223.68 cm^2
   d. 1029.22 cm^2

6. A four cylinder engine has a bore (diameter) of 7.5 cm and a stroke (height) of 1.1 cm. What is the maximum displacement (volume) of each cylinder to the nearest cubic centimetre?
   a. 49 cm^3
   b. 26 cm^3
   c. 52 cm^3
   d. 50 cm^3

7. A solvent tank measures 135 cm x 50 cm x 45 cm. To the nearest litre, how many litres of solvent will it take to fill it three-quarters full? 1 cm^3 equals 1 mL.
   a. 228 L
   b. 304 L
   c. 508 L
   d. 104 L

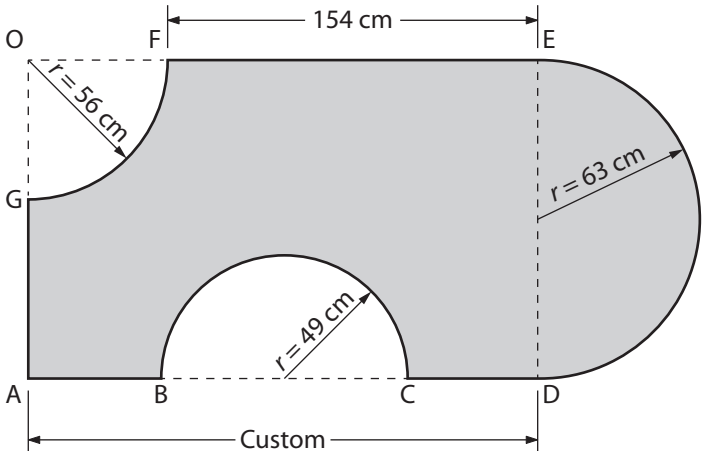
8. Find the area of the washer in the figure below with an inside diameter of 12 mm and an outside diameter of 22 mm.
   a. 85.6 mm^2
   b. 31.4 mm^2
   c. 212.08 mm^2
   d. 267.04 mm^2



9. There are two rectangular containers. The dimensions of container A are 10 m × 6 m × 5 m. The dimensions of container B are 7 m × 4 m × 8 m. Container A holds more.
   a. True
   b. False

10. A circle and a triangle with a base of 9 in. both have an area of 28.2744 in.^{2}. The diameter of the circle is greater than the altitude of the triangle.
   a. True
   b. False

11. Find the area of the metal plate shown shaded below.
   a. 26 460 cm^2
   b. 32 694 cm^2
   c. 44 725 cm^2
   d. 15 298 cm^2



12. What is the area of a triangle with base of 21.5" and an altitude (or height) of 8"?
   a. 172"^{2}
   b. 104"^{2}
   c. 86"^{2}
   d. 44"^{2}

13. A truck has a cylindrical air pressure reservoir for its air brake system with an outside diameter of 39 cm and an outside length of 84 cm. What is the internal volume of the reservoir in cubic centimetres if the cylinder walls are 0.5 cm thick?

    a. 95 266 cm^3

    b. 88 463 cm^3

    c. 100 346 cm^3

    d. 122 863 cm^3

14. A rectangular steel bar has a cross sectional area of 468 mm^2. The bar is to be reformed into a triangular-shaped bar with the same cross-sectional area. If the height of the triangle is 36 mm, what is the length of the base?

    a. 22 mm

    b. 26 mm

    c. 28 mm

    d. 32 mm

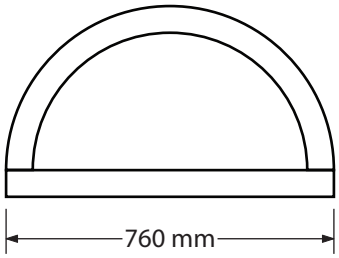
15. Calculate the cost of all moulding needed to go around the window in the figure below, if the cost per metre is $6.50. Round your answer to the nearest cent.

    a. $12.70

    b. $25.40

    c. $15.55

    d. $10.85



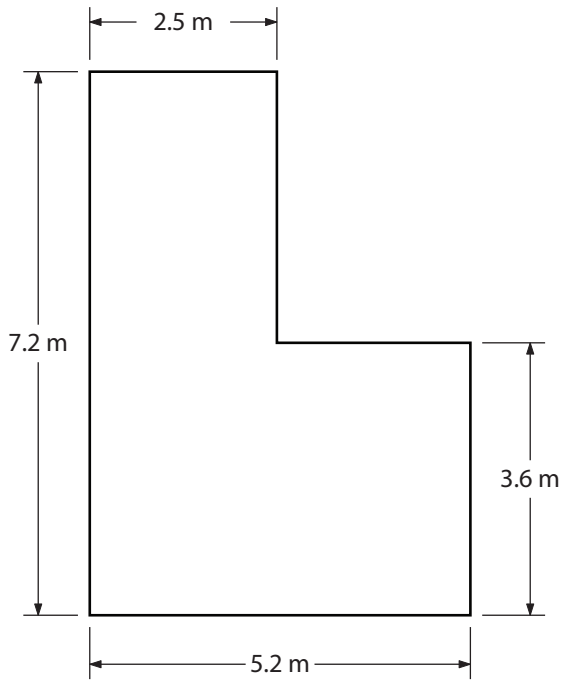
16. How much ceiling trim is required for the perimeter of the room shown in the figure below?

    a. 12.4 m

    b. 18.5 m

    c. 24.8 m

    d. 32.5 m



17. A DC generator contains four brush assemblies, with each assembly containing six brushes. The surface of each brush is 0.065 m by 0.025 m. Calculate in square metres the total surface area of all the brushes.

    a. 0.028 m^2

    b. 0.039 m^2

    c. 0.0016 m^2

    d. 0.0098 m^2

18. A cleaning tank for small parts measures 15 cm by 35 cm by 12 cm deep. The tank has a fill line 4 cm from the top. How many litres of cleaning fluid will the tank hold if filled to the fill line? (1 cm^3 = 1 mL)

    a. 6.3 L

    b. 4.2 L

    c. 3.8 L

    d. 5.8 L

19. A sheet metal worker is building a heating duct in which the rectangular vent is 15 cm high. This vent must have the same area as the rectangular opening, which is 20 cm by 30 cm. Calculate the length of the vent.

    a. 20 cm

    b. 28 cm

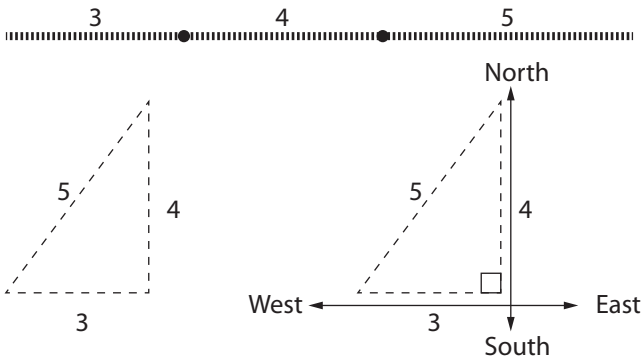
    c. 36 cm

    d. 40 cm

LEARNING TASK 8

Solve problems involving geometric shapes

When we think of geometry, we usually think about angles, circles, triangles, and squares. Geometry is simply the study of points, lines, planes, and space. As long as 4000 years ago, the Egyptians used geometrical ideas to survey land that was flooded yearly by the Nile River. They were also concerned about the orientation of their temples and had to accurately determine north and south, east and west lines. The angle between these lines had to be 90 degrees (a right angle). The Egyptians had known for some time that a 12-cubit-long rope marked out in 3-, 4-, and 5-cubit lengths would create a triangle with a 90-degree angle.



This method of creating a 90-degree angle was also known to the Chinese 3000 years ago, but it was the Egyptians who first attempted to classify and extend such geometrical ideas. Around 600 BC, the Greek Thales of Miletus visited Egypt, and when he returned to Greece, he taught geometry. Thales is credited with being the first person to use deductive logic to determine new geometrical facts.

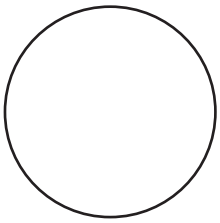
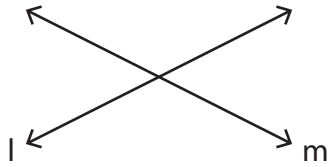
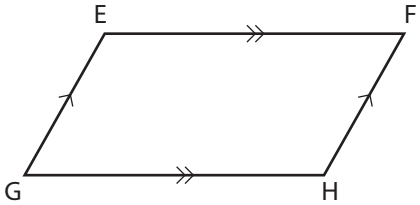
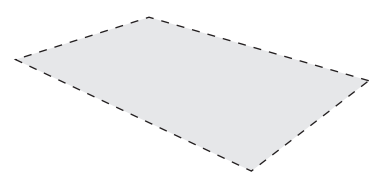
It was another Greek, Pythagoras (born around 580 BC), who forever changed the nature of mathematics. Unlike Thales, who worked only with concrete numbers and facts, Pythagoras treated geometry in a purely abstract manner. Finally, around 300 BC, Euclid, another Greek mathematician, wrote what is probably the most famous textbook of all time, *The Elements*.

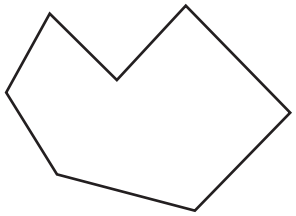
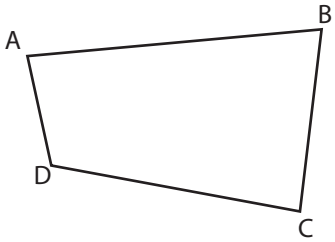
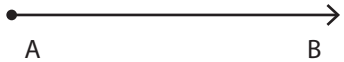
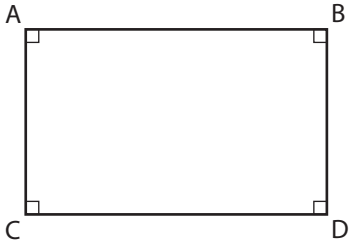
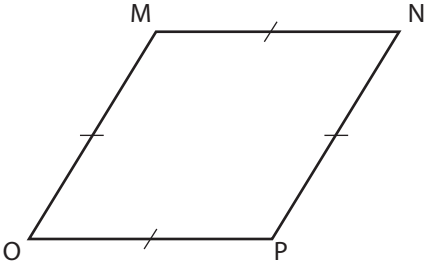
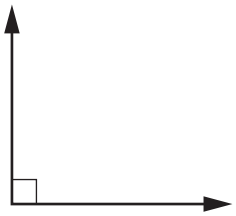
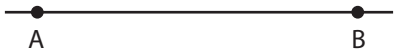
In his book, Euclid stated five "truths" or postulates about points and lines. He then went on to prove hundreds of other truths or theorems about points and lines using deductive logic. It is exactly this technique, of beginning with a few postulates and then deriving many other theorems through deductive logic, that has been at the heart of mathematics for the last 2300 years.

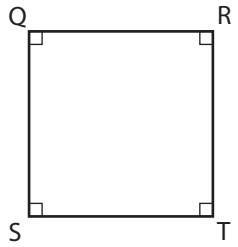
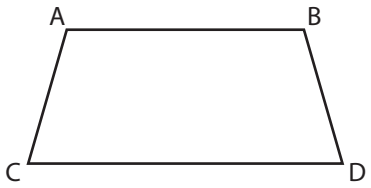
Geometry is still relevant today. Almost every machine or handmade construction involves using geometry; walls must be straight, fences must be level, shapes have to be fitted properly. Whenever triangles, rectangles, and circles are constructed, geometry is being used.

In this Learning Task we will study plane Euclidean geometry, the most useful of the geometries that deal with flat surfaces.

Key terms

Term	Definition	Symbol	Figure
Circle	A closed figure where all of the points on the figure are the same distance from a fixed point, called the *centre*.	no symbol	
Intersecting lines	Lines that do intersect and have one point in common	no symbol	
Line	A set of points extending endlessly in both directions; has length but no thickness	$\overleftrightarrow{AB}$ $\overleftrightarrow{BA}$ or *m*	
Parallel lines	Lines in the same plane that do not intersect	/ || *m* or *m* || / ↔→ ↔→	
Parallelogram	A quadrilateral with opposite sides parallel and equal in length	no symbol	
Plane	A flat surface that extends in all directions; it has length and width but not thickness. (Think of a tabletop extending endlessly in all directions.)	no symbol	
Point	A location that assumes a position but that does not extend through space to also assume dimension, magnitude, or direction	A	•A

Term	Definition	Symbol	Figure
Polygon	A closed figure made up of three or more straight line segments	no symbol	
Quadrilateral	A four-sided plane figure. (*Quad* means "four" and *lateral* means "side.")	no symbol	
Ray	A part of a line with one endpoint that extends endlessly in one direction. (Note that the starting point is always given first.)	$\overrightarrow{AB}$	
Rectangle	A parallelogram with four right angles	no symbol	
Rhombus	A parallelogram with four equal sides; its opposite sides are parallel and its opposite angles are equal.	no symbol	
Right angle	An angle that measures 90 degrees	¬	
Segment	A part of a line consisting of two endpoints and all the points in between	$\overline{AB}$ or $\overline{BA}$	

Term	Definition	Symbol	Figure
Square	A rhombus with four right angles	no symbol	
Trapezoid	A quadrilateral with one and only one pair of opposite sides parallel	no symbol	

Angles

An angle is formed whenever two rays have a common endpoint or vertex. In Figure 1, rays

$\overrightarrow{BA}$ and $\overrightarrow{BC}$ have a common endpoint, B.

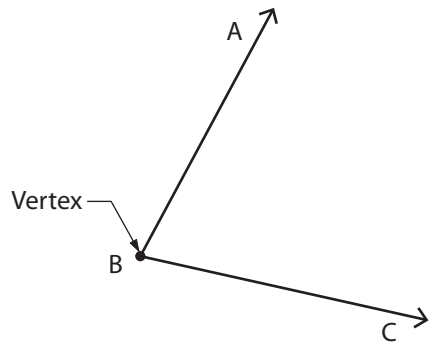


Figure 1 — Angle formed by a common endpoint

$\angle ABC$ or $\angle CBA$ or $\angle B$

The symbol $\angle$ means "angle," while A, B, and C again represent points on the angle. Note that the vertex of the angle always appears in the name as the middle letter, or, when there is no doubt about what angle is being referred to, as the only letter.

Angles can be measured in terms of the gap or spread that exists between two rays. This gap is measured in degrees, represented by the symbol °.

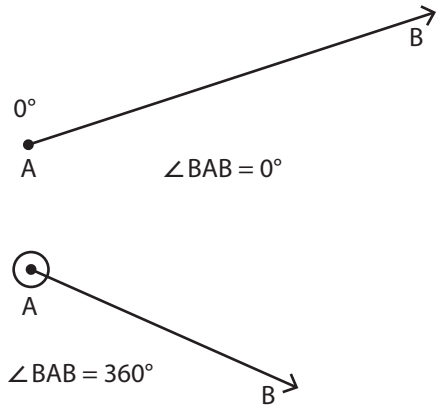


Figure 2 — Smallest and largest gaps between rays

The smallest gap that can exist between two rays is 0°. In a zero-degree angle, both rays would "lie on top of each other" (Figure 2).

The largest gap that can exist between two rays is 360°. Again, both rays of the angle are identical or "lie on top of each other" (Figure 2). The circle around point A indicates a full 360-degree rotation around that point.

Note that "∠ABC" can also mean "the measure (in degrees) on angle ABC."

Study the angles in Figure 3 and note their measures.

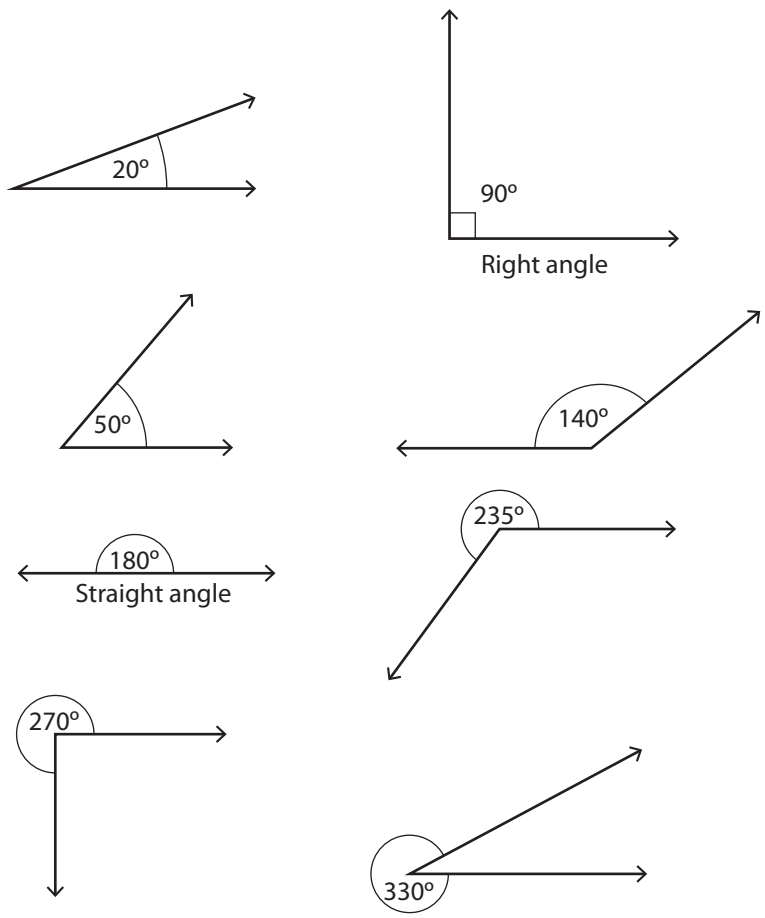


Figure 3 — Angles

Basing angular measurement on the number 360 goes back over 4000 years to the days of the Babylonians. The Babylonians used a sexagesimal number system; a base 60 system, as opposed to our base 10 system. The number 60 is good to use for a base because it can be divided evenly by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30.

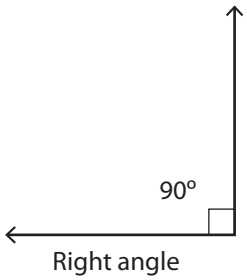


Figure 4 — Right angle

There are several things to notice about the angles in Figure 3:

- A 90-degree angle is called a *right angle* (Figure 4).
- A 180-degree angle is called a *straight angle* (Figure 5).

Figure 5 — Straight angle

Also, there are actually two different gaps or two different ways of measuring any angle. The sum of these two measures is always 360 degrees (i.e., 70 degrees + 290 degrees = 360 degrees) (Figure 6).

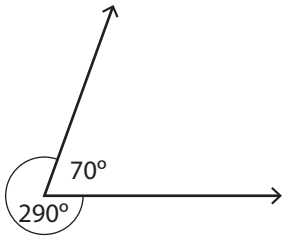


Figure 6 — Two ways to measure an angle

A device called a protractor (Figure 7) is used to measure angles in degrees. Notice that there are two scales on protractor.

- The outside scale is used for measuring angles that open to the right.
- The inside scale is used for measuring angles that open to the left.
- Some protractors may have these scales reversed.

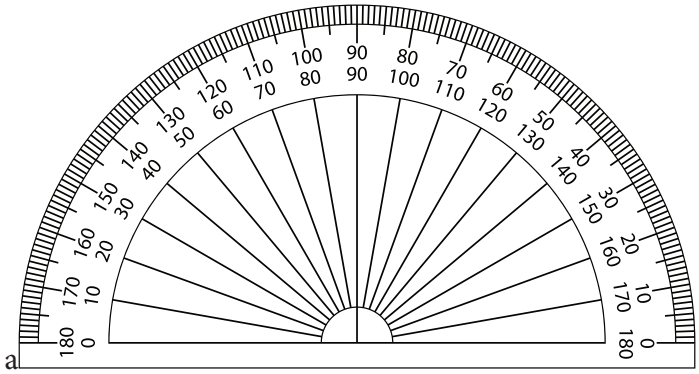
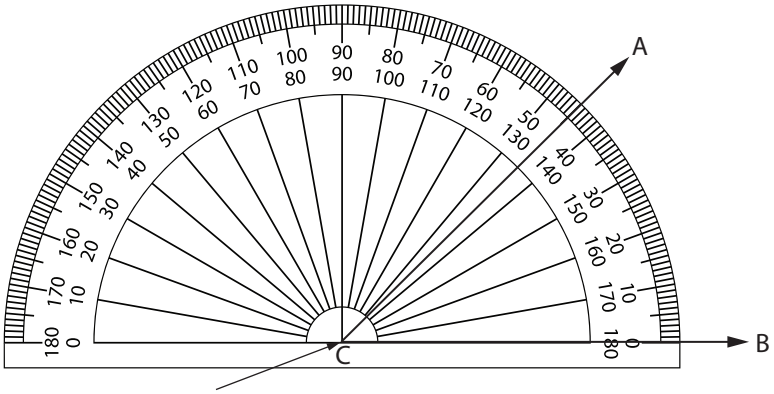


Figure 7 — Protractor

To measure an angle, place the centre mark of the protractor at the vertex of the angle and line up one of the sides on the 0° line. The other side of the angle will pass through the scales on the protractor.

Example 1
Measure ∠ACB (Figure 8).



Note that the vertex of the angle has to be located at this point, not the lower edge of the protractor.

Figure 8 — Measuring ∠ACB

Solution
Since ∠ACB opens to the right, read the measure on the scale that has zero to the lower right.

∠ACB measures approximately 45 degrees.

Example 2
Measure ∠DEF (Figure 9).

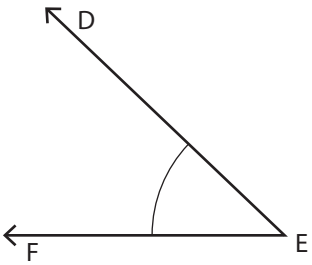


Figure 9 — Measuring ∠DEF

Solution
Since ∠DEF opens to the left, use the scale that has zero to the lower left.

∠DEF measures approximately 22 degrees. (Use your own protractor to measure the angle more accurately.)

Example 3
Measure ∠GHI (Figure 10).

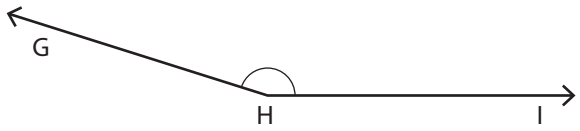


Figure 10 — Measuring ∠GHI

Solution

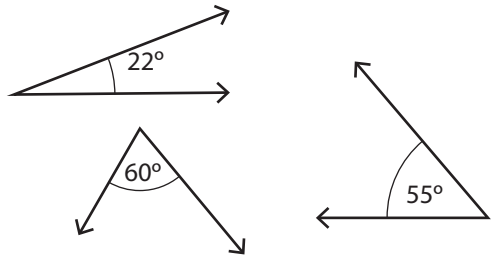
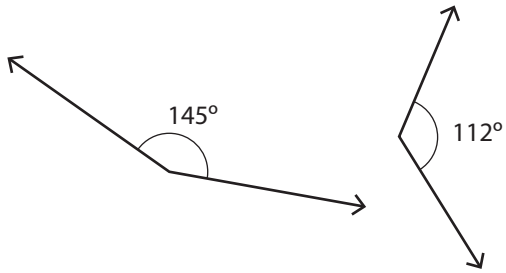
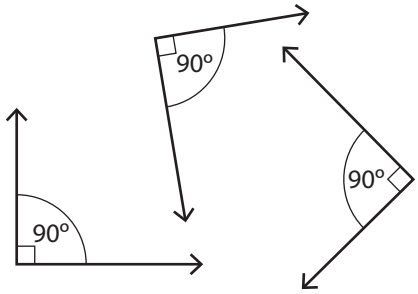
Since ∠GHI opens to the right, use the inside scale.

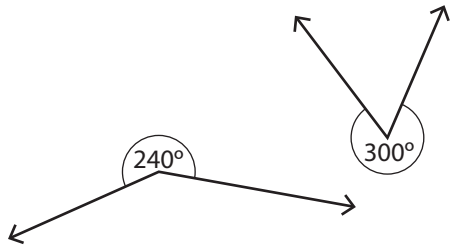
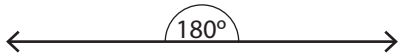
∠GHI measures about 163 degrees.

Angles: Types and properties

Classification of angles

Angles are classified according to their measurements.

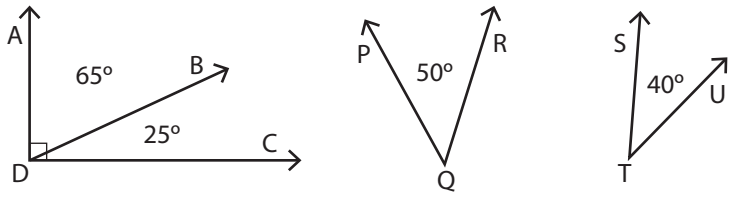
Angle	Definition	Examples
Acute angles	Angles that measure greater than 0° and less than 90°	
Obtuse angles	Angles that measure greater than 90° and less than 180°	
Right angles	Angles that measure 90°. Right angles are usually shown with a small square at the vertex of the angle.	

Angle	Definition	Examples
Reflex angles	Angles that measure greater than 180° and less than 360°	
Straight angles	Angles that measure 180°	

In this Learning Task, you will only study angles up to 360°, which is a full circle.

If the sum of the measures of two angles is 90°, then the angles are *complementary*.

For example, ∠ADB and ∠BDC are complementary and ∠PQR and ∠STU are also complementary (Figure 11).



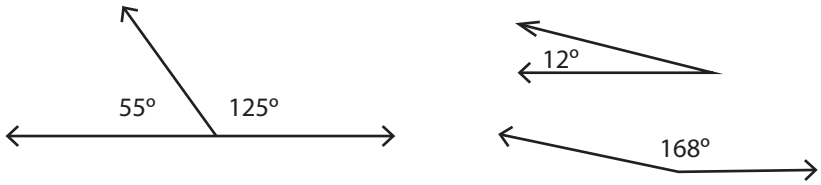
(65° + 25° = 90°) (50° + 40° = 90°)

Figure 11 — Complementary angles

Note that an angle greater than 90° cannot have a complement.

If the sum of the measure of two angles is 180°, then the angles are *supplementary*.

For example, the pairs of angles in Figure 12 are supplementary.



(55° + 125° = 180°) (12° + 168° = 180°)

Figure 12 — Supplementary angles

Two ways to help you remember which is which:

$$\begin{bmatrix} \underline{\text{C}}\text{omplementary} = 90^\circ \\ \underline{\text{S}}\text{upplementary} = 180^\circ \end{bmatrix} \text{ Since C comes before S in the alphabet}$$

$$\begin{bmatrix} \text{Com}\underline{\text{p}}\text{lementary} = 90^\circ \\ \text{Sup}\underline{\text{p}}\text{lementary} = 180^\circ \end{bmatrix} \text{ Since } 90^\circ \times 2 \text{ "p"s} = 180^\circ$$

When two lines intersect, the angles on opposite sides of the intersection point are called *vertically opposite angles*.

In Figure 13, ∠1 and ∠3 are vertically opposite. ∠2 and ∠4 are also vertically opposite.

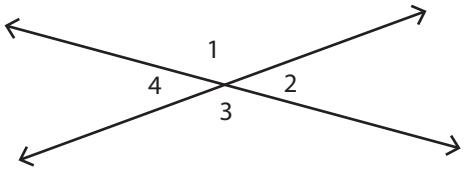


Figure 13 — Vertically opposite angles

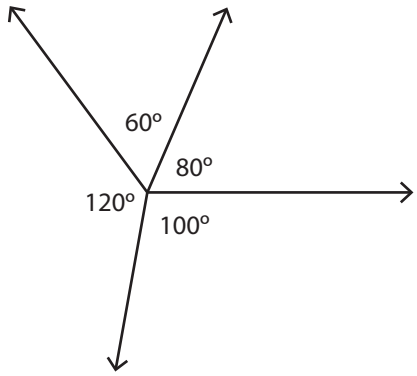
Notice that ∠1 is supplementary to ∠2 and so is ∠3. Hence ∠1 and ∠3 have the same measure.

Two angles that have the same measure are equal or are said to be *congruent*.

Vertically opposite angles are congruent.

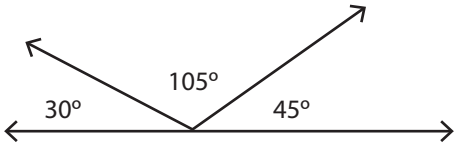
The following relationships between angles are also true.

The sum of the angles at a point is 360° (Figure 14).
The sum of the angles on the same side of a line is 180° (Figure 15).



(60° + 120° + 100° + 80° = 360°)

Figure 14 — Sum of angles = 360°



(30° + 105° + 45° = 180°)

Figure 15 — Sum of angles = 180°

Example 1

In Figure 16, m and n are lines and ∠1 = 50°. Find ∠2, ∠3, and ∠4 and state your reasons.

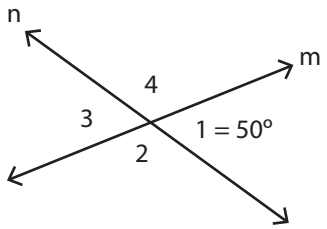


Figure 16 — Diagram for Example 1

Solution

First, write "50°" at ∠1.

	Reason
$\angle 2 = 130°$	since $\angle 1$ and $\angle 2$ are supplementary
$\angle 3 = 50°$	since $\angle 2$ and $\angle 3$ are supplementary or since $\angle 3$ is vertically opposite $\angle 1$
$\angle 4 = 130°$	since $\angle 3$ and $\angle 4$ are supplementary or since $\angle 4$ is vertically opposite $\angle 2$

Notice that in the above example you could have used the reason that $\angle 3 = 50°$ since $\angle 3$ is vertically opposite $\angle 1$. Similarly, $\angle 4$ is vertically opposite $\angle 2$.

Example 2

In Figure 17, $\overleftrightarrow{AD}$ and $\overleftrightarrow{BE}$ are lines, $\angle DFE = 30°$, and $\angle CFE$ is a right angle.

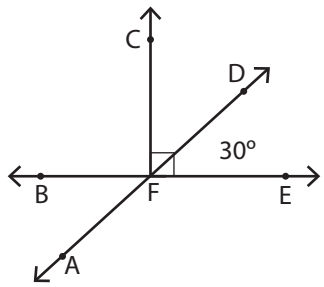


Figure 17 — Diagram for Example 2

Determine the measures of $\angle CFD$, $\angle CFB$, $\angle BFA$, and $\angle AFE$ and state your reasons.

Solution

	Reason
$\angle CFD = 60°$	$\angle CFD$ and $\angle DFE$ are complementary
$\angle CFB = 90°$	$\angle CFB$ and $\angle CFE$ are supplementary
$\angle BFA = 30°$	$\angle BFA$ and $\angle DFE$ are vertically opposite
$\angle AFE = 150°$	$\angle AFE$ and $\angle DFE$ are supplementary

Transversals and angles

A *transversal* (Figure 18) is a line that intersects two or more lines.

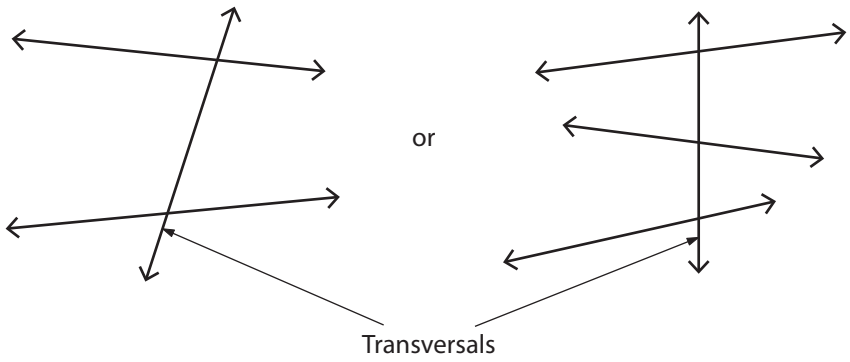


Figure 18 — Transversal

The angles formed by the transversal can be named as follows. Consider Figure 19.

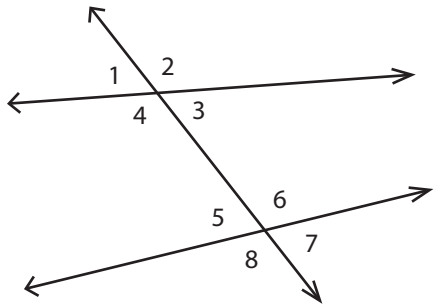


Figure 19 — Angles formed by a transversal

Interior angles

$\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ in Figure 20 are called *interior angles*. They are the four angles between the lines.

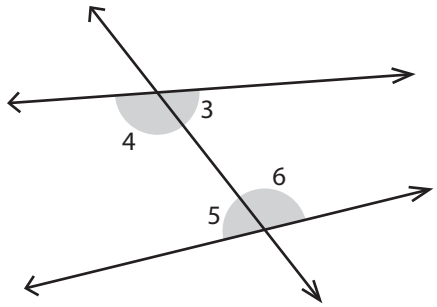


Figure 20 — Interior angles

Interior angles on the same side of the transversal

In Figure 21, ∠3 and ∠6 are interior angles on the same side as the transversal. So are ∠4 and ∠5. They are angles between the lines but on the same side of the transversal.

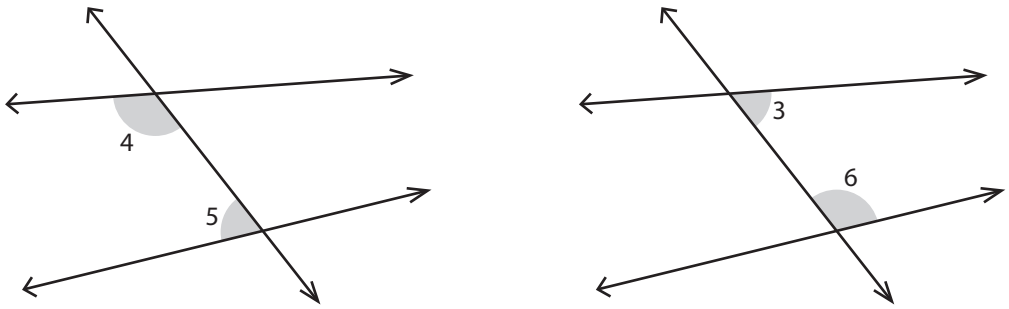


Figure 21 — Interior angles on the same side of the transversal

Alternate interior angles

In Figure 22, ∠3 and ∠5 are called *alternate interior angles*. So are ∠4 and ∠6. They are interior angles that are on opposite sides of the transversal.

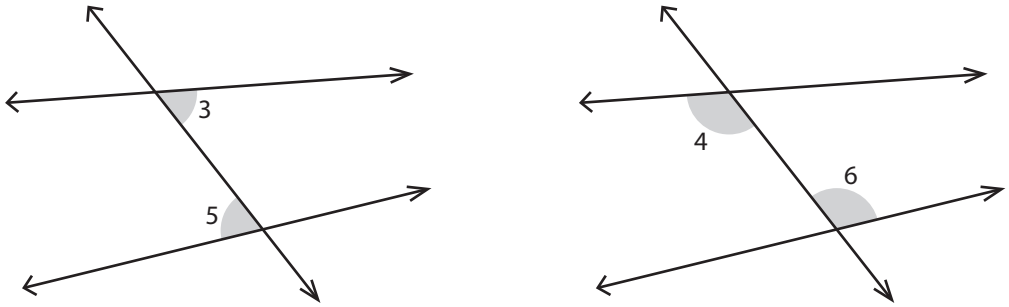


Figure 22 — Alternate interior angles

Corresponding angles

In Figure 23, ∠1 and ∠5 are called *corresponding angles*, as are ∠2 and ∠6. Other corresponding angles are ∠4 and ∠8 and ∠3 and ∠7.

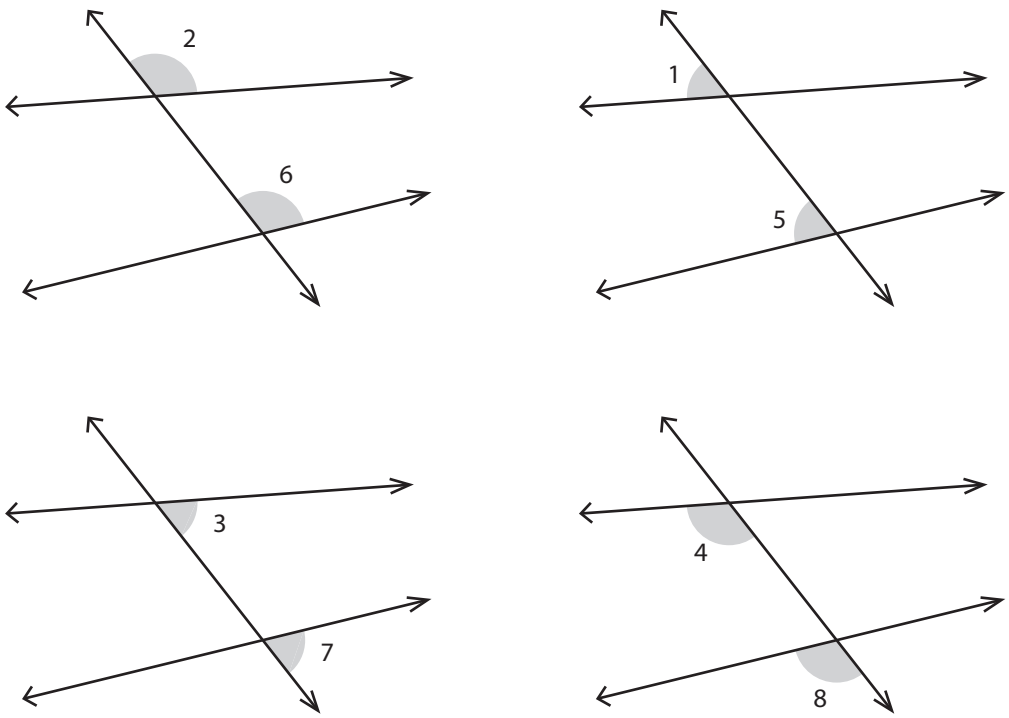


Figure 23 — Corresponding angles

Some very important relationships emerge when two parallel lines are cut by a transversal.

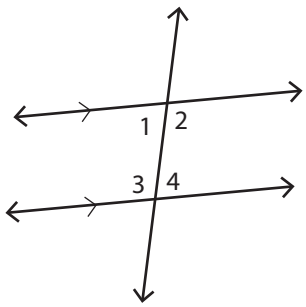


Figure 24 — Supplementary interior angles

If a transversal intersects two parallel lines. then the interior angles on the same side of the transversal are supplementary (Figure 24):

$\angle 1 + \angle 3 = 180°$
$\angle 2 + \angle 4 = 180°$

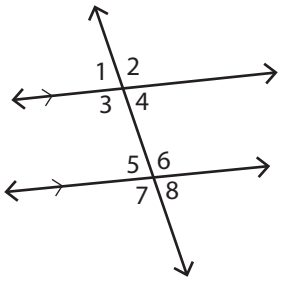


Figure 25 — Congruent alternate interior angles

If a transversal intersects two parallel lines, then the alternate interior angles are congruent (Figure 25):

$\angle 1 = \angle 4$
and
$\angle 2 = \angle 3$

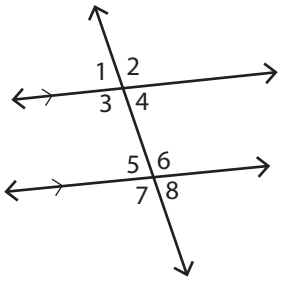


Figure 26 — Congruent corresponding angles

If a transversal intersects two parallel lines, then the corresponding angles are congruent (Figure 26):

$\angle 1 = \angle 5$ and $\angle 2 = \angle 6$
$\angle 3 = \angle 7$ and $\angle 4 = \angle 8$

The notation $\begin{matrix}\longleftrightarrow\\ \longleftrightarrow\end{matrix}$ indicates that the two lines are parallel. When you read $\angle 1 = \angle 4$, it means $\angle 1$ has the same measure as $\angle 4$. It also means $\angle 1$ is congruent to $\angle 4$.

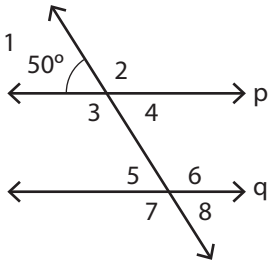


Figure 27 — Diagram for example

Example 1

p and q are parallel lines and $\angle 1 = 50°$ (Figure 27). What are the measures of the other angles? Give reasons.

Solution

	Reason
$\angle 2 = 130°$	Supplementary to $\angle 1$
$\angle 4 = 50°$	Vertically opposite to $\angle 1$ (or supplementary to $\angle 2$)
$\angle 3 = 130°$	Supplementary to $\angle 1$ (or vertically opposite to $\angle 2$)
$\angle 5 = 50°$	Corresponding to $\angle 1$ (or alternate interior to $\angle 3$)
$\angle 6 = 130°$	Supplementary to $\angle 5$ (or alternate interior to $\angle 4$ or corresponding to $\angle 2$)
$\angle 7 = 130°$	Vertically opposite to $\angle 6$ (or corresponding to $\angle 3$)
$\angle 8 = 50°$	Supplementary to $\angle 7$ (or corresponding to $\angle 4$ or supplementary to $\angle 6$)

Notice that in the example there are often several "reasons" you can use to calculate a given angle.

Closed figures (shapes)

Closed figures can be made up of curves or straight line segments, or combinations of curves and lines, such as the ones shown in Figure 28.

A closed figure begins and ends at a certain point and does not cross itself.

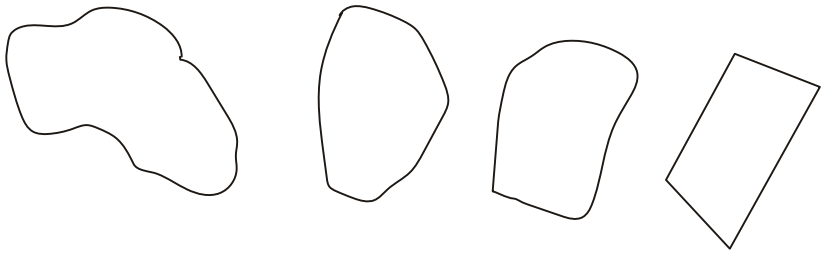


Figure 28 — Closed figures

A polygon is a closed figure made up of three or more straight line segments. The line segments are called *sides* and each meeting of two sides forms a vertex.

Polygons are classified by the number of sides they have:

# of Sides		Prefix	Figure name
3	=	tri-	triangle
4	=	quadri-	quadrilateral
5	=	penta-	pentagon
6	=	hexa-	hexagon
7	=	hepta-	heptagon
8	=	octa-	octagon
9	=	nona-	nonagon
10	=	deca-	decagon

Notice that the number of sides in a polygon equals the number of vertices (plural of *vertex*).

Triangles

The simplest polygon is a triangle (Figure 29). It consists of three sides and three angles or vertices.

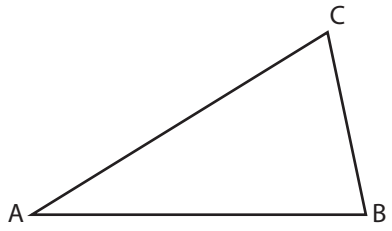


Figure 29 — Triangle ABC

Naming triangles

Naming triangles is very simple. They are named according to the three vertices by writing the letters of the vertices following the word "triangle" or the symbol Δ, which means *triangle*.

Normally, the order of giving the three vertices does not matter. For example, consider Figure 30.

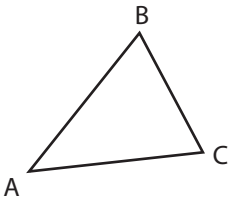


Figure 30 — Naming triangles

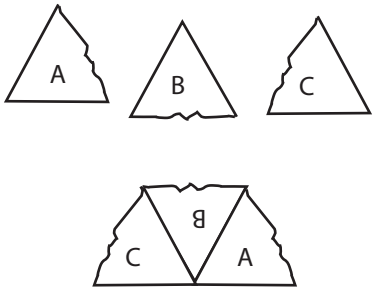
This triangle can be named:

- triangle ABC or ΔABC
- triangle BCA or ΔBCA
- triangle CAB or ΔCAB
- triangle ACB or ΔACB, etc.

Sum of angle measures in a triangle

Do the following:

1. Draw a triangle on paper. Cut out the triangle. Label it. Tear it into three pieces, each including one angle (as below).



2. Place ∠A, ∠B and ∠C together as shown.

3. Using your protractor, measure the sum of the angles. You should find that the sum of the angles is 180°.

The sum of the interior angles of any triangle is 180°.

$m\angle A + m\angle B + m\angle C = 180°$ ("$m\angle A$" means "the measure of angle A")

You can use this simple equation to find the measurement of an unknown angle in a triangle.

Example
Find the measure of $\angle X$ in the triangle shown in Figure 31.

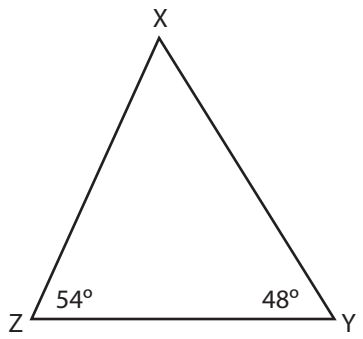


Figure 31 — Solving for $\angle X$

$m\angle X + m\angle Y + m\angle Z = 180°$

Substituting,

$m\angle X + 54° + 48° = 180°$

Solving for $m\angle X$,

$m\angle X = 180° - 54° - 48°$
$m\angle X = 78°$

Quadrilaterals

Quadrilaterals are four-sided plane figures. (*Quad* means "four" and *lateral* means "side.") The four sides are line segments.

The quadrilateral in Figure 32 can be named ABCD (or BADC or CDAB, etc.). The sides of ABCD are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$. The angles of ABCD are $\angle A$, $\angle B$, $\angle C$, and $\angle D$.

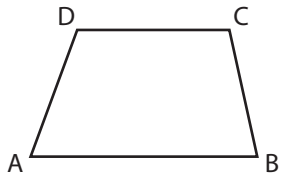


Figure 32 — Quadrilateral ABCD

Two sides are said to be opposite sides if they do not have points in common. In ABCD, sides $\overline{AB}$ and $\overline{CD}$ are opposite and $\overline{AD}$ and $\overline{BC}$ are opposite. Two sides are adjacent when they have a common endpoint. In ABCD, we say $\overline{AB}$ is adjacent to $\overline{BC}$ ($\overline{AB}$ is also adjacent to $\overline{AD}$.)

Two angles of a quadrilateral are opposite angles if they do not have a common side. In ABCD in Figure 32, $\angle A$ is opposite $\angle C$, and $\angle B$ is opposite $\angle D$.

Special quadrilaterals

There are five very special quadrilaterals.

Trapezoid

A trapezoid (Figure 33) is a quadrilateral with one and only one pair of opposite sides that are parallel.

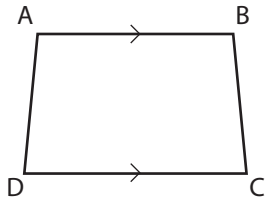


Figure 33 — Trapezoid

Parallelogram

A parallelogram (Figure 34) is a quadrilateral with opposite sides that are parallel and equal in length.

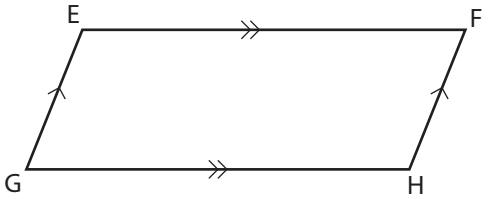


Figure 34 — Parallelogram

Because opposite sides are parallel, opposite angles in a parallelogram are equal.

Hence, $\angle E = \angle H$ and $\angle G = \angle F$.

Notice that a parallelogram is not a trapezoid, since it has more than one pair of opposite sides parallel.

Rectangle

A rectangle (Figure 35) is a parallelogram with four right angles. This means that every rectangle is also a parallelogram.

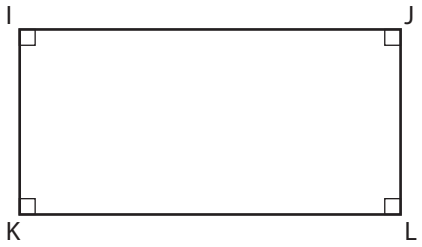


Figure 35 — Rectangle

Rhombus

A rhombus (Figure 36) is a parallelogram with four equal sides. Its opposite sides are parallel and its opposite angles are equal. Hence, every rhombus is also a parallelogram. (Is the converse of this statement true? No! Every parallelogram is not a rhombus.)

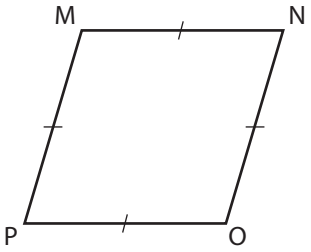


Figure 36 — Rhombus

Square

A square (Figure 37) is a rhombus with four right angles. Notice that a square is a special type of rectangle.

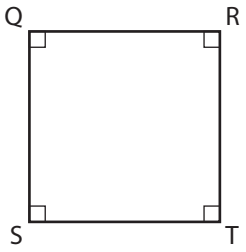


Figure 37 — Square

Circles

A circle is a closed figure where all of the points on the figure are the same distance from a fixed point, called the *centre*.

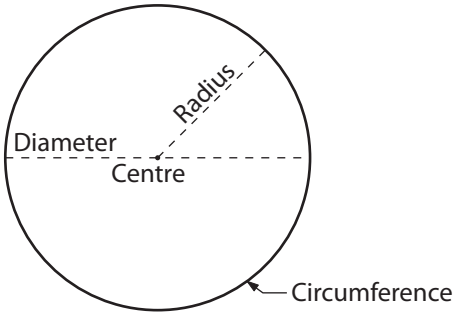


Figure 38 — Parts of a circle

Notice that the diameter of a circle is actually twice the radius of that circle (Figure 38). This can be written as a formula:

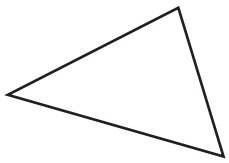
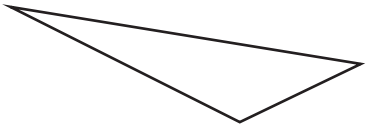
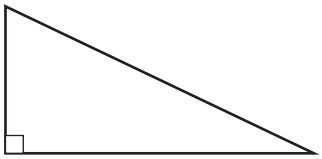
diameter = 2 × radius or $d = 2r$

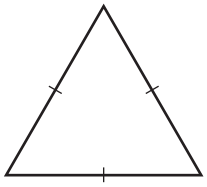
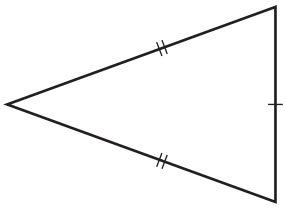
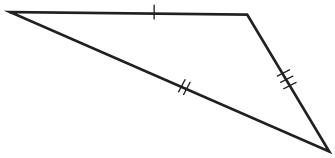
Triangles

Since a triangle is the simplest possible polygon, it is also the strongest. Triangles are used in building bridges, houses, and other structures.

Classifying triangles

Triangles can be classified in two ways: by angles and by sides.

Classification by angles		
Acute triangle	All three angles are acute (less than 90°).	
Obtuse triangle	One angle is obtuse (greater than 90°).	
Right triangle	One angle is a right angle, or 90°.	

Classification by sides		
Equilateral triangle	All three sides are equal.	
Isosceles triangle	Two sides are equal.	
Scalene triangle	No two sides are equal.	

Note: When the short hash marks made across a line segment in a polygon are the same as those on another or several line segments, it indicates that the line segments are equal.

Most triangles are scalene. That is, no side is equal to any other side and no angle is equal to any other angle.

Unless you are given more information, assume that any triangle you work with is scalene, even if the diagram appears to have equal sides or angles.

The scalene triangle is the "general" form of a triangle. Now we will look at some special triangles.

Angles in isosceles triangles

Consider the isosceles triangle in Figure 39.

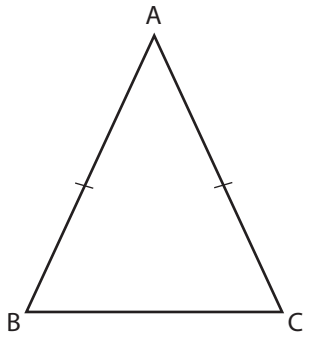


Figure 39 — Isosceles triangle

$\overline{AB} = \overline{AC}$

Which angle is across from or opposite $\overline{AB}$?

$\angle C$ is opposite $\overline{AB}$.

Measure it.

Which angle is across from or opposite $\overline{AC}$?

$\angle B$ is opposite $\overline{AC}$.

Measure it as well.

What do you find? You should find that $m\angle C = m\angle B$ ("$m\angle C$" means "the measure of angle C").

In an isosceles triangle, the angles opposite the equal sides are equal.

Angles in equilateral triangles

Consider the equilateral triangle in Figure 40.

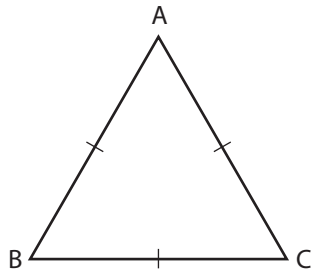


Figure 40 — Equilateral triangle

Measure ∠A.
Measure ∠B.
Measure ∠C.

What do you find?

You should find that
m∠A = m∠B = m∠C = 60°.

In an equilateral triangle, all three angles are equal and 60°.

Note: As a result of the printing process, angles in diagrams shown here may not be accurate.

Finding missing angles of triangles

These rules also can be used to find the missing angles of triangles. Consider the following examples.

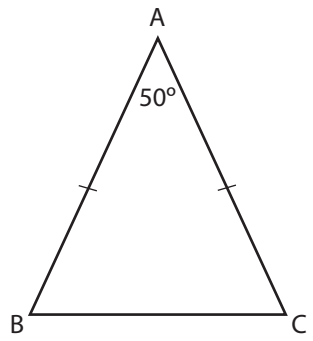


Figure 41 — Triangle for Example 1

Example 1
In the triangle in Figure 41, find m∠B and m∠C.

Solution
We know that m∠A + m∠B + m∠C = 180°.

Substituting,

50° + m∠B + m∠C = 180°
m∠B + m∠C = 180° –50°
m∠B + m∠C = 130°.

But, triangle ABC is an isosceles triangle, with $\overline{AB} = \overline{AC}$.

Therefore, m∠B = m∠C.

Substituting further,

$m\angle C + m\angle C = 130°$
$2 \times m\angle C = 130°$
$m\angle C = 65°$

and,

$m\angle B = m\angle C = 65°$.

Example 2

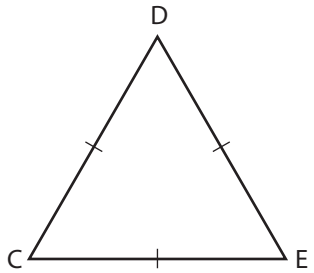


Figure 42 — Triangle for Example 2

Triangle CDE (Figure 42) is an equilateral triangle. What are the measures of its angles?

$m\angle C + m\angle D + m\angle E = 180°$

But CDE is an equilateral triangle.

Therefore,

$m\angle C = m\angle D = m\angle E$,

and substituting,

$m\angle C + m\angle C + m\angle C = 180°$.

so,

$3 \times m\angle C = 180°$ and $m\angle C = 60°$.

Similarly,

$m\angle D = m\angle E = m\angle C = 60°$.

Similar figures

The house floor plan in Figure 43 is much smaller than the house it describes, but it is similar to it. All of the angles in the floor plan are the same as the corresponding angles in the actual house.

Also, all of the dimensions in the floor plan have been shrunk by the same amount. We say that the floor plan is a *scale drawing* of the actual house.

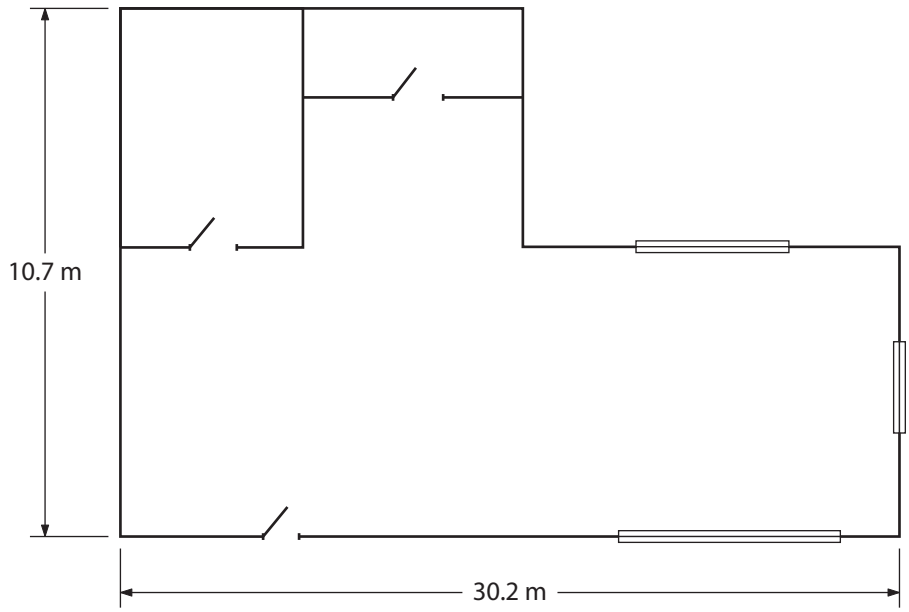


Figure 43 — Floor plan

The scale of a drawing such as this is really a ratio. For example, if the scale was 1:100, 100 cm in the actual house would show as a length of 1 cm in the scale drawing.

An unknown dimension can be found by using a proportion. For example, if the scale is 1:100 and a measurement on the drawing is 5 cm, you can find the actual house dimension with the following proportion:

$$\frac{\text{drawing } 1}{\text{actual size } 100} = \frac{5\text{ cm}}{\text{H}} \quad \text{Now cross multiply:}$$

$$\text{H} = \frac{100 \times 5\text{ cm}}{1} = 500\text{ cm (or 5 m)}$$

Similar figures **have equal angles, and their corresponding sides will be in proportion. In other words, similar figures have the same shape but may not be the same size.**

Triangles can also be similar.

Example 1

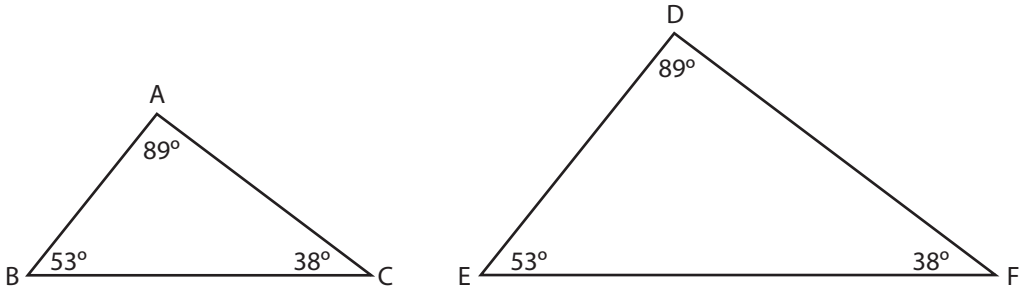


Figure 44 — Triangles for Example 1

For ΔABC and ΔDEF (Figure 44), ∠A = ∠D, ∠B = ∠E, ∠C = ∠F,

Since corresponding angles are equal, then the corresponding sides must be in proportion. The triangles ABC and DEF are similar.

ΔABC ~ ΔDEF ("~" means "similar to")

Example 2

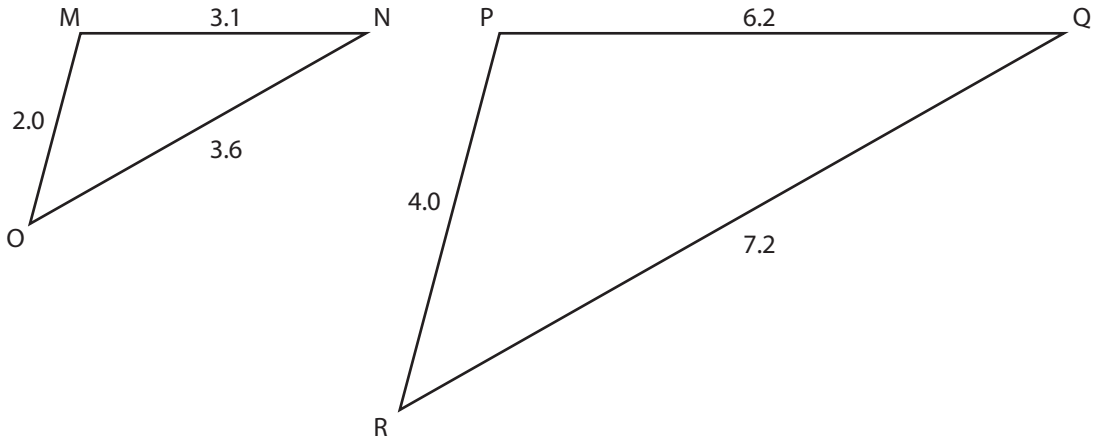


Figure 45 — Triangles for Example 2

For ΔMNO and ΔPQR (Figure 45),

$$\frac{\overline{MN}}{\overline{PQ}} = \frac{3.1}{6.2}, \frac{\overline{NO}}{\overline{QR}} = \frac{3.6}{7.2}, \text{ and } \frac{\overline{OM}}{\overline{RP}} = \frac{2.0}{4.0}$$

All of these ratios equal 1/2 or 0.5. Since corresponding sides are in proportion, the corresponding angles must be equal. Triangles MNO and PQR are similar. ΔMNO ~ ΔPQR.

Congruent triangles

When similar triangles have a 1:1 ratio or are the same size, we say they are *congruent* (Figure 46). An easier way of understanding congruency is to say that if triangles are congruent, then any one triangle can be moved onto the other in such a way that it fits exactly.

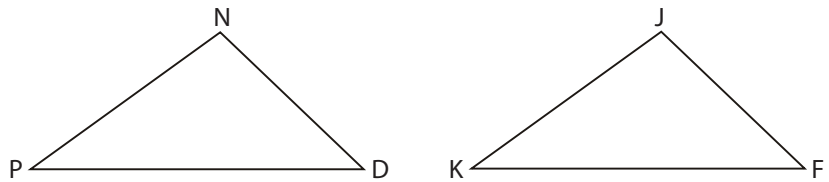


Figure 46 — Congruent triangles

Angle comparison: $\angle N = \angle J,\ \angle D = \angle F,\ \angle P = \angle K$

Segment comparison: $ND = JF,\ DP = FK,\ PN = KJ$

So, $\Delta NDP \cong \Delta JFK$ ("≅" means "is congruent to")

Note that the order of the vertices of the triangles is important.

Angle N has to be in the same position as angle J, D as angle F, and angle P as angle K.

For two triangles to be congruent they have to have equal sides and equal angles. However, to actually show that two triangles are congruent, you do not necessarily have to know that all six corresponding parts are equal (or congruent).

You can use some shortcuts that are proven geometry rules:

Side-Side-Side (SSS)
Angle-Side-Angle (ASA)
Side-Angle-Side (SAS)

1. **Side-Side-Side (abbreviated as SSS)**
   If three sides of one triangle are congruent respectively to three sides of another triangle, then the triangles are congruent (Figure 47).

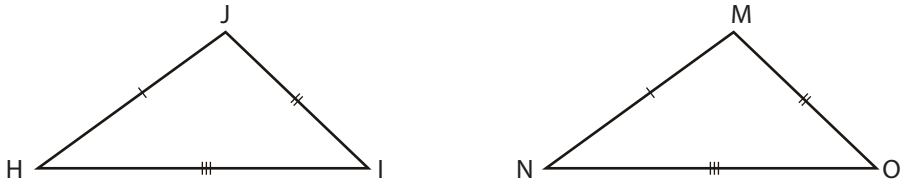


Figure 47 — Side-Side-Side shortcut

The reason for this rule is because it is impossible to construct two different triangles whose three sides are the same lengths.

2. **Angle-Side-Angle (abbreviated as ASA)**

   If two angles and the contained side of one triangle are respectively equal to two angles and the contained side of a second triangle, then the two triangles are congruent (Figure 48). ("contained" means in between)

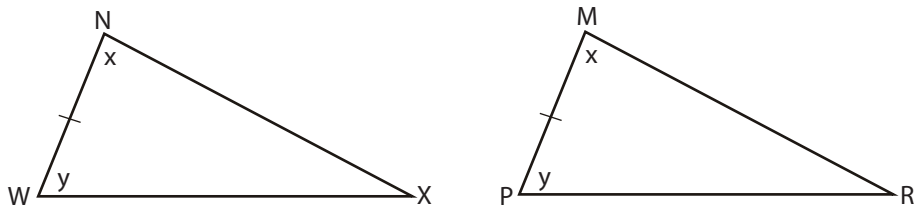


Figure 48 — Angle-Side-Angle shortcut

3. **Side-Angle-Side (abbreviated as SAS)**

   If two sides and the contained angle of one triangle are respectively equal to two sides and the contained angle of another triangle, then the triangles are congruent (Figure 49).

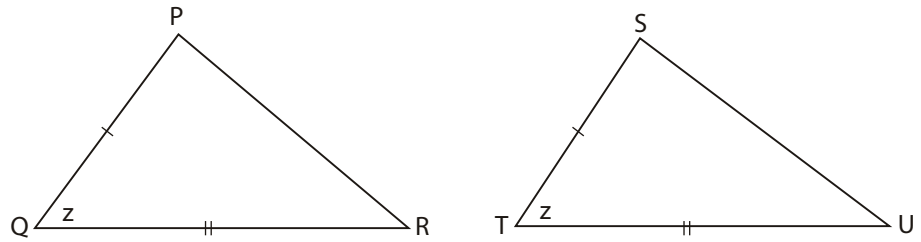


Figure 49 — Side-Angle-Side shortcut

Warning: Never assume that two triangles are congruent just because they look the same. You have to be able to determine that at least three corresponding parts of the triangles are congruent according to one of the three triangle congruency rules: SSS, SAS, or ASA.

Note that all of the above congruency rules apply to pairs of triangles that are mirror images of each other (Figure 50):

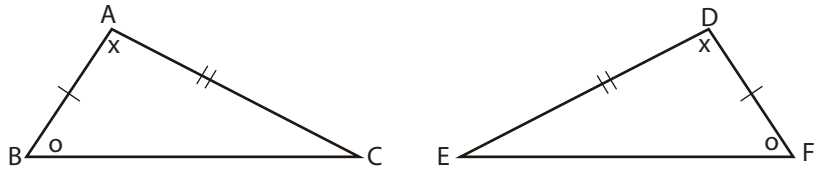


Figure 50 — SAS rule example

Since $\overline{AB} = \overline{DF}$ and $\angle A = \angle D$ and $\overline{AC} = \overline{DE}$, then the SAS (side-angle-side) rule applies, and ΔABC is congruent with ΔDFE.

There is one other important rule to know about triangles, called the *third angle equality theorem*.

This theorem states that if two angles of one triangle are congruent to two angles of another triangle, then the third angles of the triangles are congruent.

The following examples should help you see how the SSS, SAS, and ASA properties are used to prove triangles are congruent.

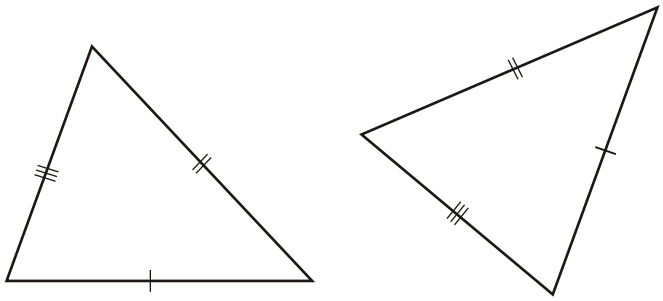


Figure 51 — Example of congruency by SSS

The two triangles in Figure 51 have three pairs of corresponding sides congruent; therefore the triangles are congruent by SSS.

Figure 52 — Example of congruency by SAS

The two triangles in Figure 52 have two sides and a contained angle congruent; therefore the triangles are congruent by SAS.

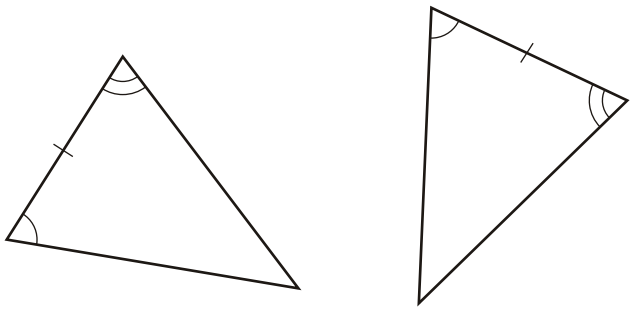


Figure 53 — Example of congruency by ASA

The two triangles in Figure 53 have two angles and a contained side congruent, therefore the triangles are congruent by ASA.

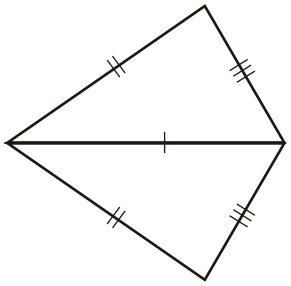


Figure 54 — Example of congruency by SSS

The polygon in Figure 54 shows two triangles with two pairs of corresponding sides congruent and a third side that is common to both triangles; therefore the triangles are congruent by SSS.

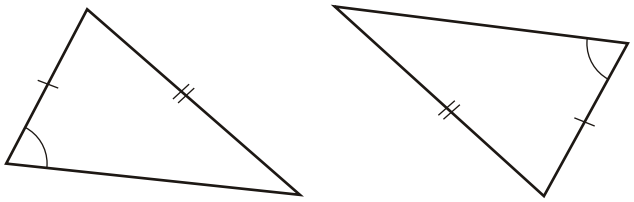


Figure 55 — Example of triangles that are not congruent

The triangles in Figure 55 are not congruent because the angle is not contained between the two sides.

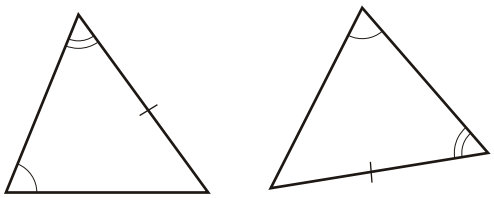


Figure 56 — Example of congruency by ASA

The triangles in Figure 56 at first glance do not look congruent, but because of the third angle equality theorem, the triangles are congruent by ASA.

The Pythagorean theorem

In mathematics, the Pythagorean theorem or Pythagoras' theorem is a statement about the sides of a right triangle. One of the angles of a right triangle is always equal to 90 degrees. This angle is the *right angle*. The two sides next to the right angle are called the *legs*, and the other side is called the *hypotenuse*. The hypotenuse is the side opposite to the right angle, and it is always the longest side. The Pythagorean theorem says that the square on the hypotenuse is equal to the sum of the areas of the squares on the legs. In Figure 57, the area of square *a* added to the area of square *b* equals the area of square *c*.

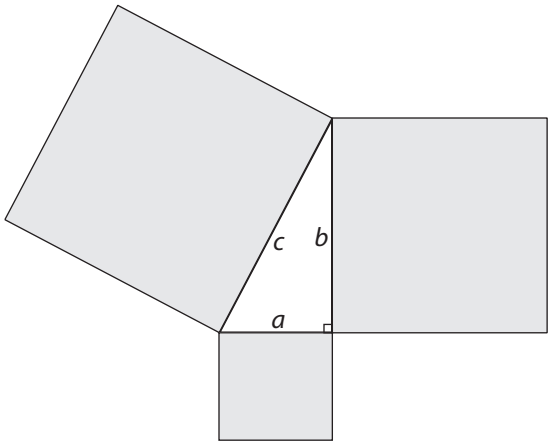


Figure 57 — Pythagorean theorem illustrated

If the lengths of the legs are *a* and *b* and the length of the hypotenuse is *c*, then:

$$a^2 + b^2 = c^2$$

In a practical application, if you know the lengths of two of the shorter sides (legs), you can solve for the hypotenuse by using the following equation:

$$c = \sqrt{a^2 + b^2}$$

If you know the lengths of one of the shorter sides and the hypotenuse, you can solve for the other shorter side by using the following equations:

$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$

Now complete the Learning Task Self-Test.

Self-Test 8

1. If two angles of a triangle add to 112°, what angle would the third be?
   a. 248°
   b. 68°
   c. 90°
   d. 48°

2. Two walls, 5' long and 12' long, meet at a corner that should be 90°. What should the distance be between the two ends of the walls?
   a. 13'
   b. 10'
   c. 17'
   d. 22'

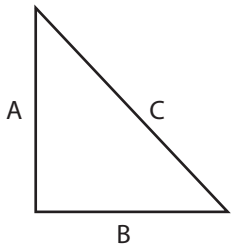
3. What is the longest side of a triangle called?
   a. Adjacent
   b. Opposite
   c. Hypotenuse
   d. Obtuse

4. What is the complementary angle to an angle of 58°?
   a. 122°
   b. 32°
   c. 62°
   d. 118°

5. If two angles are supplementary, they will equal 180°.
   a. True
   b. False

6. Guidelines for installing ceiling tiles are checked for square. A measurement of 6' is marked off horizontally on one wall and a measurement of 8' is marked off horizontally on the adjacent wall. If the guidelines are square, what should be the measurement of the line that joins the two marks?
   a. 10'
   b. 12'
   c. 14'
   d. 16'

7. Find the missing values.



A	B	C
25 mm	18 mm	_____mm
12'	_____'	16'
_____"	22"	34"
110 m	58 m	_____m
_____cm	60 cm	111 cm
54½"	61⅜"	_____"
_____mm	38 mm	66 mm

Appendices

Appendix A: Math web links

Open University sites

Numbers, units and arithmetic
This free course, "Numbers, units and arithmetic," will help you to revise whole numbers, decimals, and fractions, both positive and negative.
http://www.open.edu/openlearn/science-maths-technology/mathematics-and-statistics/mathematics-education/numbers-units-and-arithmetic/content-section-0

Ratio, proportion and percentages
http://www.open.edu/openlearn/science-maths-technology/mathematics-and-statistics/mathematics-education/ratio-proportion-and-percentages/content-section-0

Squares, roots and powers
This free course, "Squares, roots and powers," reminds you about powers of numbers, such as squares and square roots. In particular, powers of 10 are used to express large and small numbers in a convenient form, known as *scientific notation*, which is used by scientific calculators.
http://www.open.edu/openlearn/science-maths-technology/mathematics-and-statistics/mathematics-education/squares-roots-and-powers/content-section-0

Khan Academy sites

Decimals
Understanding decimals conceptually, using operations with decimals, rounding and estimating with decimals, and converting decimals.
https://www.khanacademy.org/math/arithmetic/decimals

Fractions
Understanding fractions conceptually, using operations with fractions, and converting fractions.
https://www.khanacademy.org/math/arithmetic/fractions

Negative numbers and absolute value
This tutorial will extend your understanding of numbers below 0 and thinking about "absolute" distance from 0.
https://www.khanacademy.org/math/arithmetic/absolute-value

Arithmetic properties
This tutorial will explore various ways to represent whole numbers, place value, order of operations, rounding, and various other properties of arithmetic.
https://www.khanacademy.org/math/pre-algebra/order-of-operations

Basic geometry
Get up to speed on the core foundations of geometry.
https://www.khanacademy.org/math/basic-geo

Appendix B: Metric unit symbols and relationships

Time

second	s	
minute	min	1 min = 60 s
hour	h	1 h = 60 min
day	d	1 d = 24 h
year	a	1 a = 365 d

Length

kilometre	km	1 km = 1000 m
metre	m	1 m = 100 cm
centimetre	cm	1 cm = 10 mm
millimetre	mm	1 m = 1000 mm
hectare	ha	1 ha = 1 hm^2 = 10 000 m^2

Area

square kilometre	km^2	1 km^2 = 1 000 000 m^2
hectare	ha	1 km^2 = 100 ha
square metre	m^2	1 ha = 10 000 m^2
square centimetre	cm^2	1 m^2 = 10 000 cm^2

Volume

kilolitre	kL	kL = 1000 L
cubic metre	m^3	1 m^3 = 1 kL
litre	L	1 L = 1000 mL
cubic centimetre	cm^3	1 L = 1000 cm^3
millilitre	mL	1 mL = 1 cm^3

Mass

tonne	t	1 t = 1000 kg
kilogram	kg	1 kg = 1000 g
gram	g	1 g = 1000 mg
milligram	mg	

Water

1 kL of water = 1 t of water

1 L of water = 1 kg of water

1 mL of water = 1 g of water

Prefixes

mega	M	1 000 000
kilo	k	1000
hecto	h	100
deca	da	10
deci	d	0.1
centi	c	0.01
milli	m	0.001
micro	µ	0.000001

Appendix C: Imperial (U.S.) units

Imperial (U.S.) units

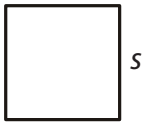
Length	Liquid Measure	Weight
1 foot (ft.) = 12 inches (in.)	1 pint (pt.) = 2 cups	1 pound (lb.) = 16 ounces (oz.)
1 yard (yd.) = 3 feet	1 quart (qt.) = 2 pints	1 ton = 2000 lb.
1 mile (mi.) = 5 280 feet	1 gallon (gal.) = 4 quarts	

Metric/imperial (U.S.) conversions

Metric to Imperial (U.S.)	Imperial (U.S.) to Metric
Length 1 cm = 0.394 inches 1 m = 39.4 inches 1 km = 0.621 miles	Length 1 inch = 2.54 cm 1 yard = 0.914 m 1 mile = 1.61 km
Area 1 hectare = 2.47 acres 1 km^2 = 0.386 square miles	Area 1 acre = 0.405 ha 1 sq. mi. = 2.59 km^2
Liquid Measure 1 L = 0.26 gallons 1 L = 4.23 cups	Liquid Measure 1 gallon = 3.79 L 1 cup = 0.24 L
Mass/Weight 1 g = 0.0353 ounces 1 kg = 2.20 pounds	Mass/Weight 1 ounce = 28.4 g 1 pound = 0.454 kg

Appendix D: Perimeter, area, and volume formulas

Square

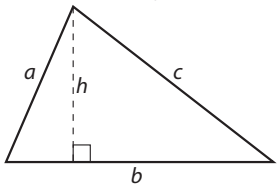


$P = 4s$
$A = s^2$

Rectangle

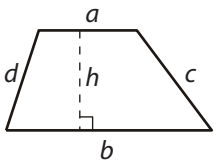
$P = 2L + 2W$
$A = LW$

Triangle



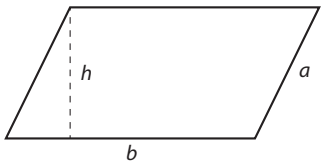
$P = a + b + c$
$A = \frac{1}{2}bh$

Trapezoid



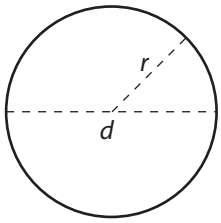
$P = a + b + c + d$
$A = \frac{1}{2}h(a + b)$

Parallelogram



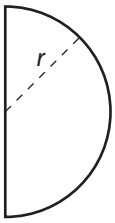
$P = 2(a + b)$
$A = bh$

Circle



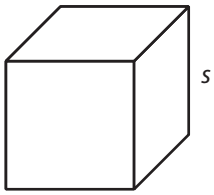
$P = \pi d$ *or* $P = 2\pi r$
$A = \pi r^2$

Semi-circle



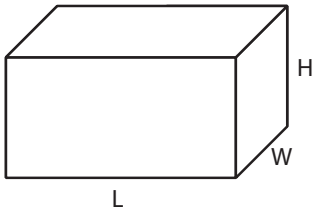
$P = \frac{1}{2}\pi d$
$A = \frac{1}{2}\pi r^2$

Cube



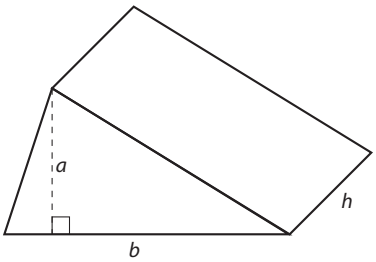
$V = s^3$

Rectangular Prism



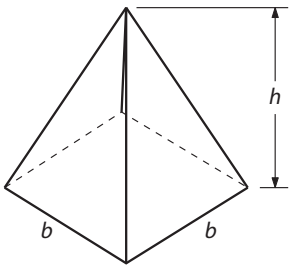
$V = LWH$

Triangular Prism



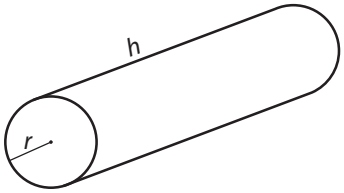
$V = \frac{1}{2}bah$

Pyramid



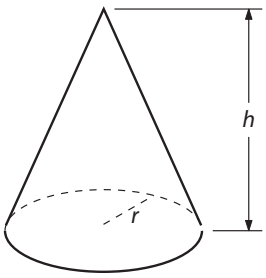
$V = \frac{1}{3}b^2h$ or $V = b^2h \div 3$

Cylinder



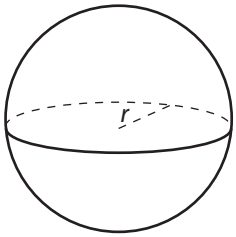
$V = \pi r^2 h$

Cone



$V = \frac{1}{3}\pi r^2 h$ or $V = \pi r^2 h \div 3$

Sphere



$V = \frac{4}{3}\pi r^3$ or $V = 4\pi r^3 \div 3$

Answer Key

Self-Test 1

1. c. 5237.021
2. b. 12 000
3. b. 2616.107
4. a. 130.545
5. c. 7.375
6. d. 149.175
7. b. 21.555
8. a. 0.003
9. a. 286
10. d. 1447.85
11. b. –74
12. c. $\frac{3}{8}$
13. a. $\frac{6}{11}$
14. b. $4\frac{1}{4}$
15. c. $\frac{87}{16}$
16. c. $\frac{25}{32}$
17. a. $\frac{7}{8}$
18. b. $4\frac{3}{8}$
19. d. $\frac{15}{128}$
20. a. 18
21. a. 6
22. d. $\frac{5}{64}$
23. b. $\frac{21}{32}$
24. a. 216.5 cm
25. c. 0.845 L
26. d. 1400
27. a. 3060
28. b. False
29. a. True
30. c. 1 m
31. a. True

32. b. 760
33. b. False
34. a. True
35. d. 33
36. b. False
37. a. 174⅙
38. a. True
39. b. 5 in.
40. b. False
41. c. 35²¹⁄₃₂"
42. a. True
43. c. 20 000

Self-Test 2

1. Perform the following metric conversions.
   a. 2360 mm
   b. 15 km
   c. 0.704 m
   d. 1200 mm
   e. 0.025 g
   f. 0.025 g
   g. 0.003 t
   h. 10 mL
   i. 17.5 cL
   j. 23 100 mL

2. Using the chart below, perform the following metric/imperial conversions. Round your answers to the nearest thousandth if necessary.
   a. 0.936 in.
   b. 45.72 cm
   c. 66.15 lb.
   d. 283.5 g
   e. 45.4 kg
   f. 9.46 L
   g. 24.84 mi.
   h. 63.103 qt.
   i. 11.37 L
   j. 11 023 lb.

3. Perform the following metric conversions.
   a. 26 800 mm
   b. 0.006 g
   c. 5 cm^3
   d. 680 g
   e. 0.153 mm
   f. 3 560 000 cm
   g. 15 kg
   h. 70 dL
   i. 0.000 013 kg
   j. 329 m

4. Using the chart below, perform the following metric/imperial conversions. Round your answers to the nearest thousandth, if necessary.
   a. 30.48 cm
   b. 75.8 L
   c. 14.7 oz.
   d. 91.5 or 91.44 cm
   e. 220.5 lb.
   f. 328 ft.
   g. 0.312 in.
   h. 131.938 km
   i. 0.454 t
   j. 0.846 qt.

Self-Test 3

1. Express the following ratios in lowest terms:
   a. 3:4
   b. 3:1
   c. 1:7
   d. 6:1
   e. 3:1

2. Write the inverse of the following ratios:
   a. 7:5
   b. 5:4

3. Find the missing term in these proportions:
   a. 14
   b. 7

4. b. 7:3

5. c. 9:5

6. b. False

7. a. 6 kL

8. c. 16

9. a. True

10. b. 2:1

11. d. 10:3

12. b. False

13. a. True

14. b. 4.636 kg
15. b. False
16. b. 36.9 V
17. In the following chart, A varies directly in proportion to B. Complete the chart.

A	7	14	**10**
B	**21**	42	30

18. a. True
19. c. 3.4 m^3
20. a. 770 min.
21. a. True
22. d. 88 421 L
23. a. 0.2 mm
24. c. 78 hp
25. a. 7 L

Self-Test 4

1. Fill in the chart below with the required numbers. Provide all fractions in lowest terms.

	Fraction	Decimal	Percent
a.	**31/50**	0.62	**62%**
b.	**2/25**	**.08**	8%
c.	2/9	**.22**	**22%**
d.	**3/40**	0.075	**7.5%**
e.	1 4/5	**1.8**	**180%**
f.	**19/20**	**.95**	95%

2. 3.78
3. 33.33%
4. 22.4
5. 60
6. b. 9.6
7. c. 45%
8. b. False
9. a. 25
10. b. 120
11. c. 158%

12. a. True
13. b. False
14. b. 20%
15. a. 15 hp
16. c. $152.99
17. c. 16
18. a. True
19. c. 193.2 mm
20. a. 3%
21. b. False
22. d. $2626.40

Self-Test 5

1. a. True
2. b. 4/25
3. c. 64
4. b. False
5. a. True
6. b. 24
7. b. 0.141
8. True
9. d. 1728
10. b. False
11. a. 2
12. c. 18
13. b. False
14. b. 0.778
15. a. True

Self-Test 6

1. b. n = 14
2. c. n = 3/8
3. a. 4
4. c. C = 131.947
5. b. $x = 32$

6. c. N = 4
7. b. N = 41
8. a. $x = 7$
9. Solve for the missing dimensions.

RECTANGLE	LENGTH	WIDTH	AREA
A	250 cm	150 cm	**37 500 cm²**
B	7.5 cm	**2.4 cm**	18 cm²
C	**1200 cm**	475 cm	570 000 cm²
D	28 cm	**23 cm**	644 cm²

10. d. P = 5280 W
11. a. t = 1.7 cm/m
12. d. P = 200 000 Pa
13. b. F = 372.6°F
14. a. 9 cm

Self-Test 7

1. c. 206 m
2. b. False
3. a. 511 sq. in.
4. d. 414 11/16"
5. b. 995.38 cm^2
6. a. 49 cm^3
7. a. 228 L
8. d. 267.04 mm^2
9. a. True
10. b. False
11. a. 26 460 cm^2
12. c. 86"²
13. a. 95 266 cm^3
14. a. 22 mm
15. a. $12.70
16. c. 24.8 m
17. b. 0.039 m^2
18. b. 4.2 L
19. d. 40 cm

Self-Test 8

1. b. 68°
2. a. 13'
3. c. Hypotenuse
4. b. 32°
5. a. True
6. a. 10'
7. Find the missing values.

A	B	C
25 mm	18 mm	**30.8 mm**
12'	**10.58'**	16'
25.92"	22"	34"
110 m	58 m	**124.35 m**
93.39 cm	60 cm	111 cm
54½"	61⅜"	**82¹⁄₁₆"**
53.96 mm	38 mm	66 mm

The British Columbia Open Textbook Project

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